Resonance & Reactance

Although it's R&R. it's not rest and recuperation - at least not just yet - instead Geoffrey Billington G3EAE is offering us an introduction to tuned circuits principles. It's guaranteed maths-free (well, very nearly).

When alternating voltage is applied to a circuit, it will take a short time, typically a few cycles, for all currents and voltages throughout the circuit to settle into a steady repetitive rhythm with the same frequency as the source. This article will be limited to dealing with circuits of this nature.

It's a fortunate and convenient fact that if the applied voltage is sinusoidal, (varying smoothly with time) like the waveforms shown in **Figs. 1, 2** and **3**, and if the circuit contains only combinations of resistors, inductors and capacitors, **then all the currents and voltages throughout the circuit will rapidly settle into sinusoidal waveforms also**.

The purpose of Fig. 1 is simply to clarify future references to 'voltage' and 'current'.

Current Variations

The curves shown in Fig. 1, show voltage and current variations for a resistor. The current peaks at 5A and the volts at 10V. Not only at the peaks, but at every instant the voltage reading is double the current reading, which means that the resistor obeys Ohm's Law and its resistance is 10/5 = 2Ω.

The voltage and current are tied together so to speak, they peak together and pass through zero together. The voltage across a resistor is always in phase (in step) with the current through it.

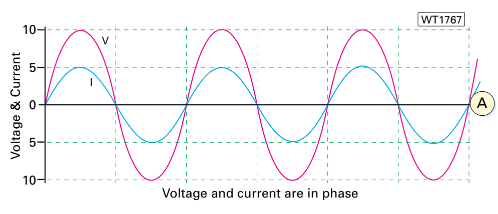

Fig. 1: Plotting current and voltage waveform for a 10V peak sinusoidal signal applied to a purely resistive load of 2Ω.

Reactance

Both inductors and capacitors exhibit 'impedance', but which is called reactance. Inductors and capacitors have a restricting effect on the flow of an alternating current, but unlike resistors, their impedances are critically dependent upon the frequency, although in opposite ways.

Even though in Fig. 2 and Fig. 3, the ratios of peak voltage to peak current are equal to two, it would be entirely wrong to call these ratios 'resistance'. This would imply the voltage and current are in a constant ratio at every instant, which they are not.

It's possible to refer to the ratio of peak voltage to peak current as an impedance. However, a new name is required to indicate that we are dealing with a special type of impedance for which voltage and current are in quadrature. The term we use 'reactance' - or more correctly 'the reactance at the operating frequency' - is usually denoted by the symbol 'X' and in Fig. 2 and Fig. 3, the reactance, X, is said to be 2Ω in each case.

At very low frequencies, an inductor behaves like a length of conducting wire, but its impedance increases as the frequency is increased. Conversely, the impedance of a capacitor is extremely high at very low frequencies, behaving almost like an open circuit, but it falls virtually to zero at sufficiently high frequencies.

When the frequency is kept constant, the behaviour of an inductor or capacitor may seem superficially similar to that of a resistor. The peak voltage remains in a constant ratio to the peak current, no matter what changes are made to the circuit.

Separated In Time

However, when considering reactance, voltage and current do not peak together, but are separated in time by one quarter of a cycle (termed a 90° phase shift). For the inductor, the voltage peaks one quarter of a cycle before the current, i.e. **V leads I**, while for the capacitor, **V lags I**.

In spite of the fact that resistance and reactance are both measured in ohms, **they are different** and **may never** be simply added together when both occur in the same circuit. In fact, numerical values of reactance are often prefixed by the symbol 'j' to ensure that they are not confused with resistances.

In Fig. 2, the inductive reactance shown would usually be written as j2Ω. However, in Fig. 3, the capacitive reactance shown would be written as -j2Ω. The 'label' 'j' (the symbol has a lot more applications than can be described here) is used to indicate that the voltage leads the current by one quarter cycle whilst '-j' shows that it lags by the same amount.

No Power Dissipated

There is one more important fact which can he shown to follow

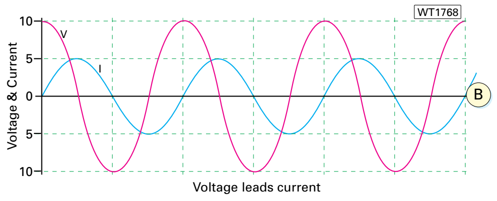

Fig. 2: The current and voltage waveforms for a 10V peak sinusoidal signal applied to a purely inductive load of 2Ω at the operating frequency. The voltage waveform is said to lead the current waveform by 90°. (See text for more detail).

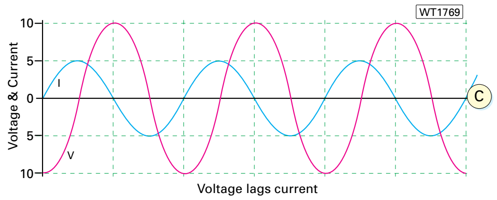

Fig. 3: The current and voltage waveforms for a 10V peak sinusoidal signal applied to a purely capacitive load of 2Ω at the operating frequency. The voltage waveform is said to lag the current waveform by 90°.(See text for more detail).

when volts and current are in quadrature: **no power is dissipated**. Inductors and capacitors (ideally) do not waste energy or get hot in operation.

In actual fact, inductors, which usually consist of a length of coiled wire, have a small, but not negligible resistance as well as reactance, but this will be ignored for the present. Our inductors will remain perfect or 'ideal' until further notice!

Now to look at circuits, such as the box shown in **Fig. 4**, which can represent a resistor, inductor or capacitor, or any combination of them. The box forms part of a circuit in which a sinusoidal alternating current flows.

The remainder of the circuit is unimportant and isn't shown. The meters for measurements of voltage and current are assumed to be sophisticated instruments, able to sample the voltage or current at any point in the cycle without loading.

Real Circuit

Now to a 'real' circuit as shown in **Fig. 5**, a series combination of a capacitor 'C', an inductor 'L' and a resistor 'R' connected to a variable frequency source. Forget about the resistor for the present and concentrate on the voltmeters V_C, V_I and V_O.

The surprising result is that V_O is always equal to the **difference** between V_I and V_C, bearing out what was stated earlier: V_I and V_C always act in opposite directions when the same current flows through both L and C.

Across the inductor, V_I leads the current by one quarter cycle and across the capacitor, V_C lags the current by one quarter cycle. This gives a full half cycle phase difference between the two voltage components. At every instant, these two voltages act in opposite directions (they subtract), though their resultant voltage combination will depend on the frequency of operation.

Series Resonance

Series resonance is a special state as it occurs at the frequency at which the reactance of the capacitor (X_C) and the reactance of the inductor (X_I) are equal. Imagine what happens when the frequency is set to a very low value and then gradually increases. At the low frequency X_C will be high and X_I will be low. At a sufficiently high frequency, this state of affairs will be reversed.

At some intermediate frequency though, the two reactances will be equal. **The frequency at which this occurs is termed the 'resonant' frequency of the combination**. As the same current flows through both inductor and capacitor, the voltages V_I and V_C will be equal as well as opposite (phase), which means that **at every instant, V_O, the voltage across the combination, will be zero**! When this happens, the combined effect of inductor and capacitor is to act like a short length of connecting wire, the only thing which limits the current flowing under this condition is the resistor.

In the real world, every inductor has a resistive component in addition to its reactance, **but its effect is just the same** as if it was a separate series resistance as shown in Fig. 3. If this were absent, the supply would effectively be short circuited.

When the frequency is varied, the current drawn from the supply will peak at the resonant frequency. At this frequency, the voltage generator would only 'see' the resistance, which would be the only thing limiting the current, the inductor and capacitor having neutralised one another (tuned each other out).

It's worth mentioning that at resonance, the voltages across the inductor and capacitor, **although equal and opposite** may be much larger than the applied supply voltage. (And both rise to a maximum at the resonant frequency).

At frequencies other than at resonance, the circuit behaves as a 'complex impedance' with both a resistive and a reactive component. At very low frequencies, the reactance of the capacitor would predominate, at very high frequencies, the reactance of the inductor predominates.

The arrangement of Fig. 5 is known as a series tuned circuit, or sometimes, an 'acceptor circuit', because it readily passes or accepts currents at the resonant frequency. Now let's look at another case!

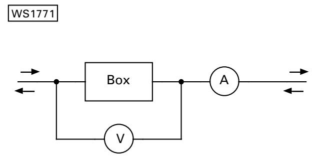

Fig. 4: The basic measurements are made of the voltage across and the current through a circuit represented by the box. (See text for more detail).

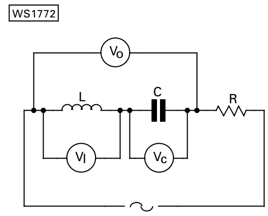

Fig. 5: Considering the separate constituent parts of a series tuned circuit. The resistor R is the calculated sum of all the resistive losses of the complete circuit. (See text for more detail).

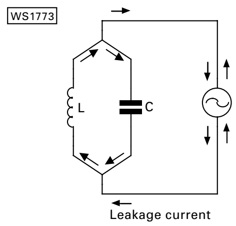

Fig. 6: In a parallel tuned circuit, using ideal components the leakage current could drop to zero, while huge currents could flow back and forth between the capacitor and inductor. (See text for more detail).

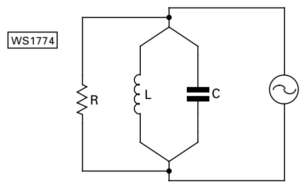

Fig. 7: With non-ideal components in the parallel tuned circuit, the resistive losses may be represented as a high value resistor in parallel with the capacitor and inductor. (See text for more detail).

Parallel Resonance

The other case of resonance is that of a parallel tuned circuit, as shown in **Fig. 6**. This is probably the more familiar circuit. In this case, the alternating voltage is applied equally to both L and C. In this circuit, the voltage (level and phase) must be the same across both L and C. But the **currents** through L and C will flow in opposite directions in order to maintain the correct cycle phase differences with the voltage.

With parallel resonant circuits, the current supplied by the generator is the **difference** between the currents through L and C. Again at the resonant frequency (when $X_I = X_C$) the currents will be equal - but in opposition. So, they'll be effectively sourcing each other, requiring no current from the generator once the system has settled down.

At resonance, the equal and opposite currents in the two arms simply form a closed circulating system, with the current surging clockwise and anticlockwise around the LC combination. An ideal parallel tuned circuit at resonance, would draw no current from the supply, its impedance would be therefore be infinite, behaving as an 'open circuit'.

Fortunately, the impedance of a real-life parallel tuned circuit at the resonant frequency is not too difficult to predict. I explained earlier that the impedance of a real inductor may be represented by **a pure inductance in series with a small resistance.** It may equally well be represented as a **pure inductance in parallel with a high resistance**.

As an extreme example, let's now take the case of the perfect inductor. We could say either that the series resistive component is zero or alternatively that the parallel resistive component is infinite. Things are a bit more complicated with a real inductor, but even so, any real parallel tuned circuit can be represented as shown in **Fig. 7**, i.e. as an ideal tuned circuit with a high resistance connected in parallel.

At the resonant frequency, this ideal tuned circuit would behave as an open circuit so only the high parallel resistance would be 'seen' by the generator. The behaviour of parallel tuned circuits is often illustrated by a set of resonance curves as shown in **Fig. 8**. The curves show how the complex impedance 'Z' - typically measured in kilohms - varies with frequency.

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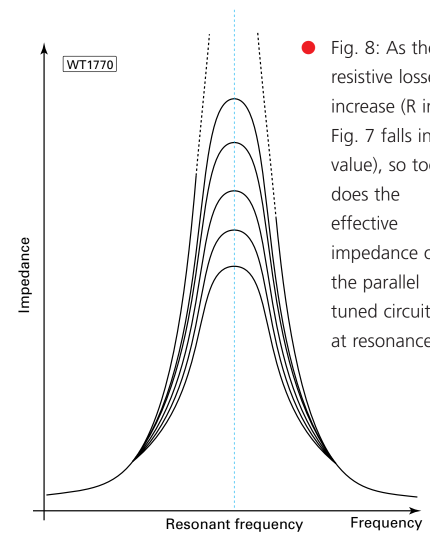

Fig. 8: As the resistive losses increase (R in Fig. 7 falls in value), so too does the effective impedance of the parallel tuned circuit at resonance.

Become Infinite

For a perfect resistance-free parallel tuned circuit, the impedance at resonance would become infinite and the resonance curve would rise to an infinitely high spike. The result of increasing the residual series resistance is the same as **decreasing** the equivalent parallel resistance, causing the spike to flatten into a lower and lower 'bump'.

Although resonance curves are useful, they do not show the composition of the impedance. This increases (at low frequencies) from an almost pure, low inductive reactance through to a pure high resistance at resonance, then decreases to an almost pure, low, capacitive reactance at high frequencies. Apart from at the resonance condition, the impedance will always be 'complex', i.e. it will contain both resistive and reactive components.

The numerical value (in ohms) of a complex impedance is once again equal to the ratio of peak volts to peak current, but a knowledge of this figure is of limited value unless the phase lag or lead (any value between one quarter cycle and zero) is also known. (This state is beyond the scope of this article to pursue this topic).

Finally, it's interesting to imagine what would happen if we could really connect up a perfect tuned circuit to a supply at the resonant frequency. The circuit would eventually draw no current from the supply, which could then be disconnected. The circulating currents would continue oscillating indefinitely in the isolated circuit!

With a real circuit however, the oscillations will die out rapidly due to losses. A similar train of damped (decaying) oscillations at the resonant frequency could be obtained without using any source of alternating voltage, by simply discharging a charged capacitor through an inductor. It's always a good thing to have a working understanding of **why** things happen. So, the following non-mathematical explanations may be of some use.

Capacitive Quadrature

Once explained, it should be fairly easy to see why any capacitor produces a capacitive quadrature shift when a capacitor is being repeatedly charged and discharged by an alternating current. All you need to do is think about an instant when the capacitor has just finished charging and is about to start discharging. **At this instant, the capacitor's charge is at maximum and therefore so is the voltage across it**.

However, the current, at this instant of charge maximum, is on the point of changing direction - **it's flowing neither one way or the other**, so it is momentarily zero. This shows that the voltage across the capacitor is at maximum when the current is passing through zero, i.e. they are in quadrature.

Inductive Quadrature

Inductive Quadrature on the other hand, is a little more awkward to explain. With this phenomena an inductor only produces a voltage when the current through it is changing, the greater the rate of change, the higher the voltage it produces. If you look at any of the current graphs you can see that the current maximum rate of change occurs where the current graph crosses the horizontal axis, i.e. as the current passes through zero. **It's at this point that the voltage peaks**.

Once again, current and voltage are in quadrature. (I warned you that this explanation was not as straightforward). Unfortunately, these two explanations of quadrature do not explain why inductors and capacitors produce **opposite** quarter cycle shifts, although it's possible to give fairly simple working explanations.

Capacitors & Generator

Suppose that a capacitor was connected directly to the output of a signal generator whose peak voltage remained constant, but whose frequency could be varied. **Whatever the frequency**, alternate current surges would charge up the capacitor to the same peak voltage and this would always require the same quantity of electric charge to circulate.

At a higher frequency, less time is available for the circulation of the charge so, the current must be larger. The reactance of a capacitor decreases when the frequency is increased.

Now let's consider inductors, but with them connected to a signal generator with a constant output current. At low frequencies current has a long time to build up. And this slow buildup equates to a low voltage (and low impedance).

As the frequency is increased, the peak current is still the same as before, but it has to change in a shorter time, thereby producing a higher voltage. The reactance must have therefore increased. Admittedly, the above explanations could benefit with a more detailed explanation, but they are basically valid!

Actual Units

One thing which has been ignored so far, is the contribution to reactance by the actual units of the inductance or capacitance as measured in Henrys (for inductors) or Farads (for capacitors). Once again, inductors and capacitors behave in opposite ways, the larger the inductance in Henrys the larger the reactance at any given frequency, whilst for capacitors, the larger the capacitance in Farads, the smaller the reactance.

Calculating Reactances

If you wish to find the approximate reactance of a known inductance or capacitance at some particular frequency, you will find the charts given in the RSGB and ARRL handbooks and elsewhere, which will enable you to do this. If you want a more accurate value, the textbooks give the formulas, but unfortunately, these are awkward to use because of the inconvenient size of the units involved.

If you already know the reactance of 'X1' of any given capacitance C1 or inductance L1 at any known frequency (F), it is easy to find the reactance (X) of any other capacitance (C) or inductance (L) at any other frequency (F) using the formulas given below.

The great beauty of these formulas is that they will work with **any** units. For example, capacitance could be in picofarads or microfarads, inductance could be in millihenrys or microhenrys, frequency could be in MHz or kHz...or whatever you like, so long as you are consistent.

For capacitances

$$X = X_1 \left(\frac{C_1}{C}\right)\left(\frac{F_1}{F}\right)$$

For inductances

$$X = X_1 \left(\frac{L}{L_1}\right)\left(\frac{F}{F_1}\right)$$

A useful set of figures for use with picofarad capacitors is the fact that a capacitance of 100pF has a reactance of 160Ω at a frequency of 10MHz. As an example, let's suppose you want to find the reactance of a 50µF capacitor at a frequency of 5MHz.

$$X = 160\left(\frac{100}{50}\right)\left(\frac{10}{5}\right) = 640\Omega$$

You can work out your own figures for use with the inductance formula, given that the reactance of 1µH at a frequency of 1MHz is 6.28Ω.

Note: The figures quoted above, of 160Ω and 6.28Ω, are close approximations rather than the more accurate versions that may be needed for some problems. For the capacitor a more accurate figure is 1000÷2π or 159.164Ω. Similarly, the figure 6.28Ω for inductive impedance is an approximation for 2π (6.2828Ω). Challenging perhaps...but useful knowledge!

PW