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Simple Locus Curves in the Smith Diagram

Impedances of antennas, inputs and outputs of amplifiers, components, etc. are often displayed according to real and reactive components as a function of frequency in one plane as a "locus curve". The "Smith Diagram" is especially suitable for such display. This can be considered to be a "bent" complex plane. The capacitive or inductive behaviour of the test object, return loss, matching range, and standing wave ratio as a function of frequency, and more, can be taken from the position and the frequency-dependent run of the impedance locus curve. It is possible to determine the L, R, and C of the two-pole and to select suitable compensation or matching measures.

Figures 1a and 1b show simple locus curves. In both cases, the standardized impedance Z_{st} is 50 Ω (reference impedance), since this is the standard impedance used in conjunction with most measuring systems in the VHF and UHF

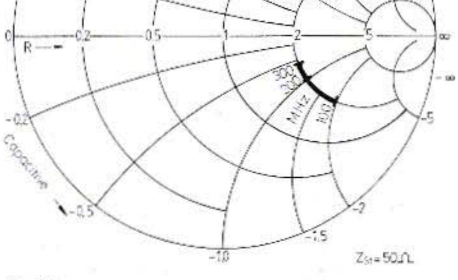


Fig. 1a:
Locus curve of a series-circuit of R and C

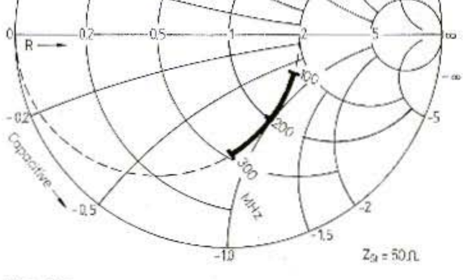


Fig. 1b:
Locus curve of a R/C-parallel circuit

range. According to the locus curve shown in **Figure 1a**, it will be seen that the measured object only represents a **series-circuit** comprising an ohmic resistance and a capacitance:

- The real component (R' = 2) is constant for all frequencies of the locus curve, which is only valid for series circuits; after destandardization: R = 2 x 50 Ω = 100 Ω.
- The reactive component is reduced on increasing frequency, and seems to be inversely proportional to the frequency, since the spacing is reduced (−2; −1; −0.66), which can only be valid for a capacitance.
- The locus curve is within the negative half of the plane, which is a further indication of a capacitive construction! $X_c'_{100\,MHz} = -2$, or destandardized $X_{c\,100\,MHz} = -2 \times 50\,\Omega = -100\,\Omega$, which corresponds to approx. 16 pF!
  $X_c'_{200\,MHz} = -1$, $X_{c\,200\,MHz} = -1 \times 50\,\Omega = -50\,\Omega$. Minus sign means "capacitive".

As can be seen in **Figure 1b**, the locus curve does not possess a simple systematic behaviour between 100 MHz and 300 MHz. Both the real and reactive components of the individual locus-curve points are different! Digital impedance meters would indicate the real and reactive components for the various frequencies according to the table given in **Figure 2** (factor "j" for "reactive component", and sign "−" for "capacitive"). The user will firstly determine from the numerical values that the test object represents a series circuit with a reactive component, whose value is frequency-dependent. This is nothing special.

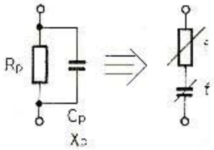
Furthermore, he will determine that the real component is frequency-dependent, and this is rather peculiar when one does not consider the skin effect!

Actually, the test object is a simple, parallel circuit comprising an ohmic resistance (of course with a constant value), and a capacitance. The following is to show how such a locus curve is made. Firstly let us examine why a **parallel connection of R_p and C_p** must be present:

- The curve of the impedance as a function of increasing frequency turns towards the zero-point of the impedance plane; in the case of $f \rightarrow \infty$, it seems that the impedance decreases towards zero, which is the case with a parallel capacitance: it will short out the ohmic resistance at higher frequencies!
- It seems on studying the locus curve that a real component remains at very low frequencies, which means that the parallel capacitance C_p is not effective, since the locus curve will exhibit a real component at $f \rightarrow 0$, in other words for DC-voltages. In the example shown in Figure 1b this will be the value R' = 2, or destandardized R_p = 2 x 50 Ω = 100 Ω.

However, a prerequisite of the previous assumptions is that the considered part of the locus curve is **part of a semicircle**, which can be assumed due to the position of the three curve points for 100, 200 and 300 MHz. Only then can one assume a **RC-parallel circuit**!

A **RL-parallel circuit** would also result in a semicircular line in the resistance plane, however, in the upper resistance plane. In this case, the semicircle will be in the direction of the zero point for $f \rightarrow 0$ (the inductance will short out the parallel ohmic resistance for DC-voltages, f = 0), and at $f \rightarrow \infty$ it will go through R_p (the parallel inductance is high-impedance at higher frequencies, and



f MHz	Real component	Reactive component
100	80 Ω	− j 40 Ω
200	50 Ω	− j 50 Ω
300	30 Ω	− j 46 Ω

Fig. 2: Table of the real and reactive components of the equivalent circuit

therefore does not have any effect on the ohmic impedance).

There are two possibilities to obtain the coordinate values R_s and X_s of the equivalent series-circuit as a function of frequency from a parallel connection of R_p and X_p, in order to display it in a locus curve as shown in Figure 1b: Either with the aid of equations, or by using a graphic method.

The **calculation equations** are:

$$R_s = \frac{1/R_p}{(1/R_p)^2 + (1/X_p)^2},$$

$$X_s = \frac{1/X_p}{(1/R_p)^2 + (1/X_p)^2}.$$

In the case of a parallel capacitance, insert $X_p = -1/\omega C_p$, and with a parallel inductance $X_p = +\omega L_p$. NOTE: The sign "−" of the capacitive, and "+" of the inductive reactive impedance indicates the phase between voltage and current!

Example: The following is assumed:
$R_p = 100\ \Omega$, $C_p \approx 8$ pF. This results in:
$X_p = -1/(2 \times \pi \times 10^8\ \text{Hz} \times 8 \times 10^{-12}\ \text{F}) = -200\ \Omega$.
This is valid for 10^8 Hz ≙ 100 MHz.

The following can be determined from this:

$$R_s = \frac{^1/_{100}}{(^1/_{100})^2 + (^1/_{(-200)})^2}\ \Omega \approx 80\ \Omega,$$

$$X_s = \frac{^1/_{(-200)}}{(^1/_{100})^2 + (^1/_{(-200)})^2}\ \Omega \approx -40\ \Omega.$$

This allows the series values given in the Table in Figure 2 to be checked.

The **graphic method** is more of interest here. This is made as follows:

The real and reactive impedance R_p and X_p are standardized with Z_{st} and from their reciprocal values conductance and susceptance are obtained: G' and B' (the apostroph is used to show that these are standardized values!). These values are now inserted into a separate Smith diagram (**Figure 3, above**) which then serves as "conductance plane". The actual **advantage of such a diagram** is: The coordinates R_s' and X_s' of the equivalent series circuit are symmetrical around the center (≙ 1) of the Smith diagram. This means

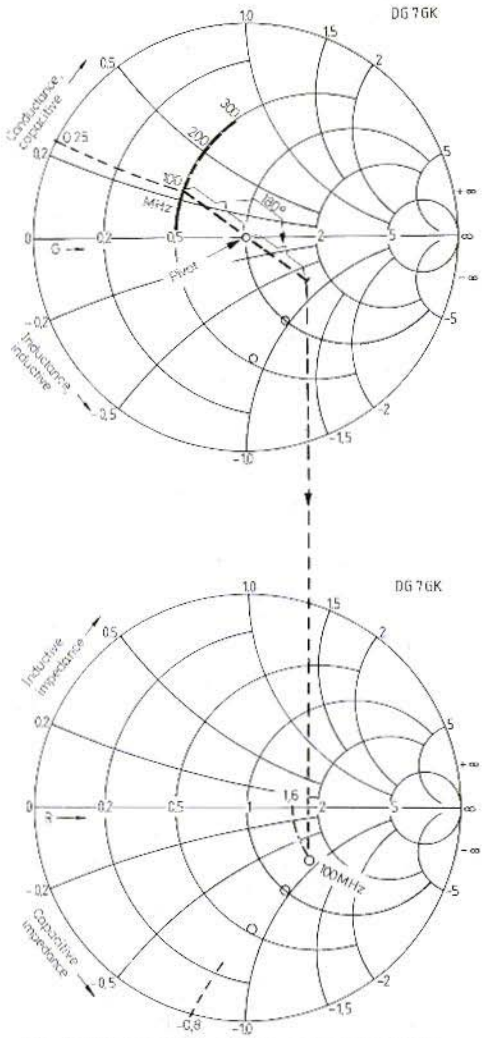


Fig. 3: Smith diagram: above conductance plane, below impedance plane

that if one rotates the point for the conductance value around the center of the diagram by 180° and inserts the new position into a second Smith diagram which is used in the resistance plane (see **Figure 3, below**), the coordinates of the transposed point will directly give the (standardized) real and reactive components of the equivalent circuit without having to use the extensive formulas given in the previous section.

Example: The conductance and susceptance values of a parallel circuit comprising 100 Ω and a capacitance of 8 pF are to be inserted in the conductance plane of a Smith diagram for a frequency of 100 MHz. The coordinates of the real and reactive impedance of the equivalent series circuit are to be obtained from the point transposition! Standardized impedance is $Z_{st} = 50\ \Omega$.

Solution: (also see Figure 3, above and below):
$R_p = 100\ \Omega$, $X_{pc} = -1(2 \times \pi \times 10^8 \times 8 \times 10^{-12})\ \Omega = -200\ \Omega$.
$R_p' = 100\ \Omega / 50\ \Omega = 2$.
$X_{pc}' = -200\ \Omega / 50\ \Omega = -4$.

The standardized conductance values G' are obtained from the reciprocal values of the standardized impedances as follows:

$G' = 1/2 = 0.5$ and $B_c' = -(1/-4) = +0.25$.

NOTE: The change of sign on transposing the reactive impedance to the susceptance, and vice versa is necessary because the sign indicates the angle of the voltage reffered to current with respect to resistance, whereas in the case of conductance values the sign indicates the angle of the current with respect to the voltage. This means that a negative susceptance value B_L will result from a positive, parallel, inductive reactive impedance X_{pL}!

The standardized conductance values, for example 0.5 and +0.25, are now inserted into the conductance Smith diagram as shown in Figure 3, above. The symmetrical line is now drawn through the center point of the diagram to obtain the symmetric reciprocal point. The symmetrical point will have the same spacing to the center of the diagram, since the reflection factor does not change due to this (theoretical) conversion process. It is now necessary for this point to be transferred to a second Smith diagram (Figure 3, below) which is the impedance plane. The coordinates of this point in the impedance plane are the standardized values of R_S and X_S of the equivalent series circuit, and are $R_s' = 1.6$ and $X_s' = -0.8$. After destandardization one will obtain the following impedance values that are given in the Table of Figure 2 for 100 MHz:

$R_s = 1.6 \times 50\ \Omega = 80\ \Omega$, $X_s = -0.8 \times 50\ \Omega = -40\ \Omega$.

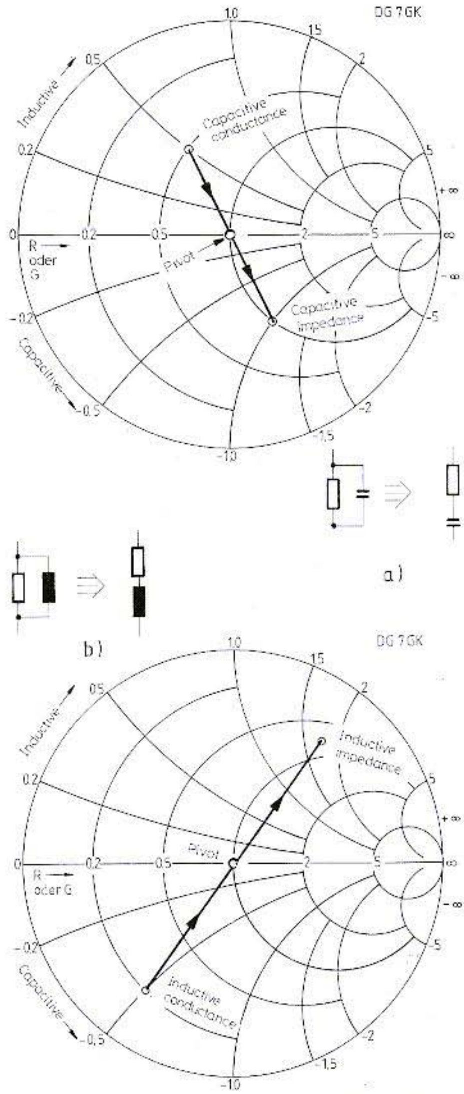


Fig. 4: Conversion (inversion) of a parallel circuit ($G = 1/R_p$; $B = -1/X_p$) into a series-circuit (R_s;X_s)
a) Parallel circuit of R and C
b) Parallel circuit of R and L

It is now possible to insert the values for 200 MHz and 300 MHz into the conductance plane (Smith diagram, Figure 3, above) in a similar manner, and to transpose them into the impedance plane of the Smith diagram by rotating them by 180°. The interconnection of these points results in the locus curve given in Figure 1b, which also shows graphically that the circuit is a parallel connection.

At first, it is recommended that separate Smith diagrams are used for conductance and impedance planes. It can be advantageous to have one of the two in the form of transparent paper or foil. Lateron, after sufficient experience has been gained, it will be possible for the user to insert the standardized conductance values, and read out the standardized impedances using the same diagram, and to interpret the coordinates correctly either as conductance coordinates or as impedance coordinates (see **Figure 4a and 4b**). Figure 4b shows that the same is valid for parallel circuits with an inductive component, as was shown for such with a capacitive component; it is only necessary to exchange the sign of the reactive component.

As has been previously mentioned, it is possible with sufficient knowledge and correct interpretation of the locus curve to find suitable transformation and matching networks. The capacitive component of the circuit shown in Figure 1a, for instance, can be compensated for by using a series-connected inductance (see **Figure 5a**). This results in a series-resonant circuit, and one will see that the compensation is only valid exactly for one frequency on the locus curve, (this frequency is 100 MHz in the case of Figure 5a). In a Smith diagram for impedances, this will mean that the locus-curve point to be compensated is shifted towards the positive plane due to the series-inductance. The new locus curve will, however, remain on the original coordinate of constant real component. In the example given in Figure 1a, this is the standardized real component R' = 2, or destandardized R = 100 Ω. The ohmic component is thus identical after compensation. A matching to another ohmic impedance, for instance 50 Ω, cannot be achieved by series-connection of a further reactive element. This is only possible with the aid of a parallel capacitance in conjunction with a series inductance; however, further details regarding this are not to be discussed here.

In the case of a circuit as shown in Figure 1b and Figure 2, it is possible to achieve not only a compensation of the reactive component, but also a transformation to another real impedance, such as 50 Ω, with the aid of a series-connection of an inductance (see **Figure 5**). This, of course, is also only valid for one discrete frequency, however, the large spacing of the real and reactive components is shifted relatively near to the center of the diagram 1 ≙ 50 Ω, in the vicinity of this frequency, as can be seen in Figure 5b.

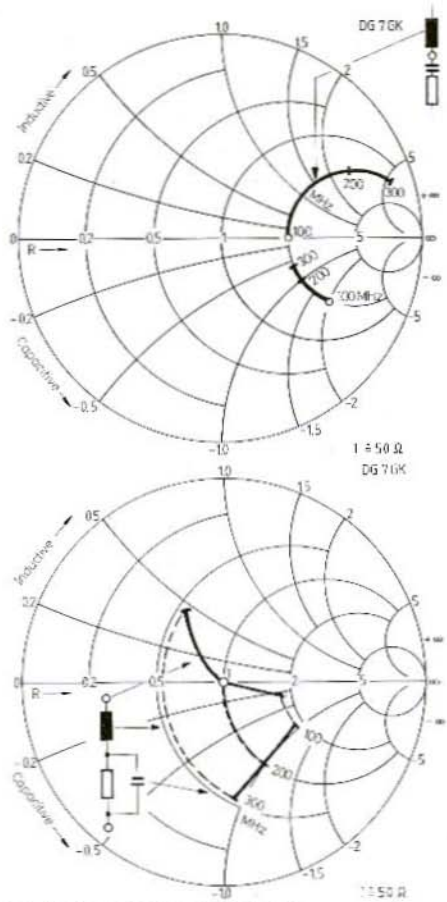


Fig. 5: Effect of a series-inductance
a) Compensation of series-C by series-L at a discrete frequency
b) Matching to 1 ≙ 50 Ω at one frequency using a series-inductance L

The selection of the value of such reactive elements for transformation and compensation networks is not the task of this article.