Analysis of a Variable Attenuator Using a 3 dB Quadrature Coupler

Variable terminations on coupler ports provide good return loss performance and allow simplified control circuitry

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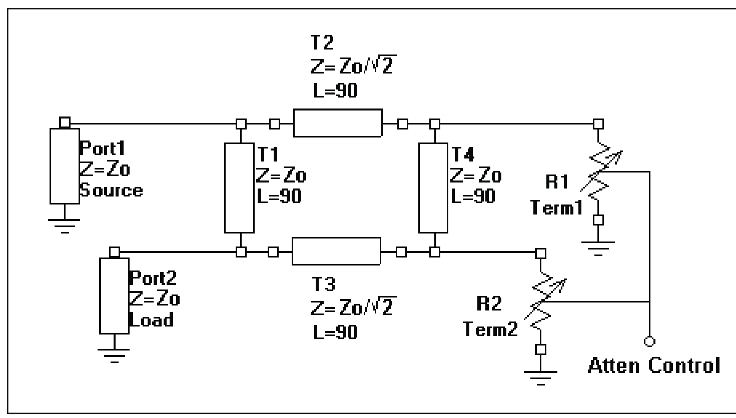
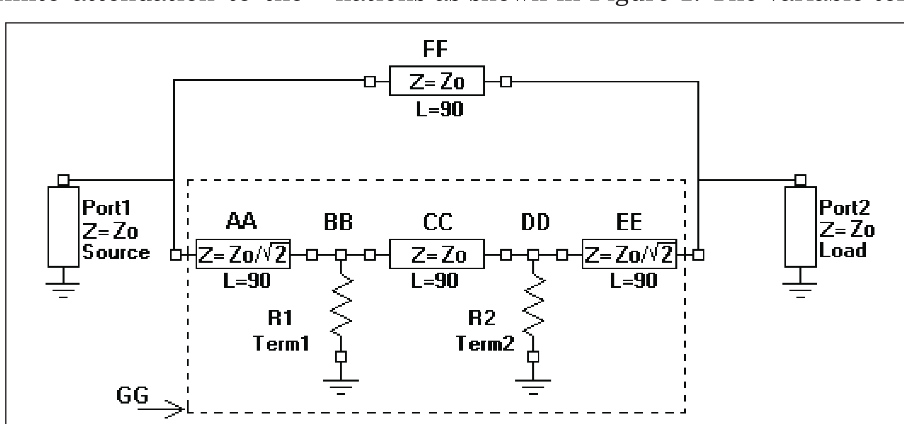
This article describes the performance of a variable attenuator formed from a 3 dB quadrature coupler with tracking variable terminations on the coupled ports. To help understand its operation, we must recognize that the attenuator can be analyzed using simple two-port techniques. The resulting *s*-parameters are obtained for the general case where the terminating impedances are unequal and for the case where the terminations track each other perfectly. We find that the effects of unequal terminations are departures from ideal non-reflective input/output port impedances and a variance in attenuation from the ideal.



▲ **Figure 1. Configuation of the coupler as an attenuator.**

The idea of creating a variable attenuator from a lossless 3 dB quadrature coupler is inspired by the fact that the isolated, or decoupled, port presents infinite attenuation to the input signal when the coupled ports are terminated with reflection coefficients equal to zero ($\Gamma_c = 0$). It has zero attenuation of the input signal when the coupled ports are entirely reflective ($|\Gamma_c| = 1$) and equally terminated. Between these two extremes ,the input port to isolated port transmission is proportional to the magnitude of the reflection coefficient of the (tracking) coupled ports. Thus a variable attenuator is implemented by using a 3 dB quadrature coupler with the isolated port designated as the output port while terminating the coupled ports with equal and variable terminations as shown in Figure 1. The variable ter-



▲ **Figure 2. Coupler schematic redrawn to emphasize two-port structure.**

minations could be a pair of identical co-packaged PIN diodes for current-controlled attenuation or a pair of identical MESFETs for voltage-controlled attenuation [1].

Analysis

A common method of analysis is to employ even-odd mode theory [2]. However, two-port analysis techniques will be used here. The author believes this approach to be simpler and more direct for this particular configuration. The following analysis assumes lossless transmission line elements.

The typical 3 dB branch line quadrature coupler of figure 1 has been redrawn in Figure 2 to emphasize the two-port structure. Each two-port element has been assigned a label from AA to FF. Label GG identifies the two-port resulting from the cascade of two-ports AA through EE. The final two-port representation of the entire network is GG in parallel with FF.

The procedure for determining the *s*-parameters for the entire network is as follows:

1. Write the *y*-parameters for two-port FF in matrix form.
2. Write the ABCD parameters for two-ports AA through EE in matrix form.
3. Find the ABCD parameter matrix GG by multiplying matrices AA, BB, CC, DD, and EE together in that order.
4. Convert GG from ABCD parameters to Y parameters.
5. Add the *y*-parameter matrices FF and GG.
6. Convert the resulting *y*-parameter matrix from step 5 into *s*-parameters. This *s*-parameter matrix represents the *s*-parameters for the entire network.

Following the above procedure, we can obtain the analysis that begnis on this page and continues on following pages.

This case represents the ideal variable attenuator. Since $s_{11} = s_{22} = 0$, the input and output ports are perfectly matched regardless of the attenuation. The magnitude of s_{21} and s_{12} is equal to the magnitude of the reflection coefficient Γ_c. Since $|\Gamma_c|$ can take on any value from 0 to 1, the input to output port transmission ($|s_{21}|$) can be made to have any value ranging from complete rejection to complete transmission.

Figure 3 shows the attenuation in dB for real values of Z_L from 0 to twice Z_0. At $Z_L = Z_0$ the attenuation becomes infinite and the slope of

Determination of network *s*-parameters

Step 1:

$$[FF]_Y = \begin{bmatrix} 0 & \dfrac{j}{Z_0} \\ \dfrac{j}{Z_0} & 0 \end{bmatrix}$$

Step 2:

$$[AA]_{ABCD} = [EE]_{ABCD} = \begin{bmatrix} 0 & \dfrac{jZ_0}{\sqrt{2}} \\ \dfrac{jZ_0}{\sqrt{2}} & 0 \end{bmatrix}$$

$$[BB]_{ABCD} = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{R1} & 1 \end{bmatrix}$$

$$[CC]_{ABCD} = \begin{bmatrix} 0 & jZ_0 \\ \dfrac{j}{Z_0} & 0 \end{bmatrix}$$

$$[DD]_{ABCD} = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{R2} & 1 \end{bmatrix}$$

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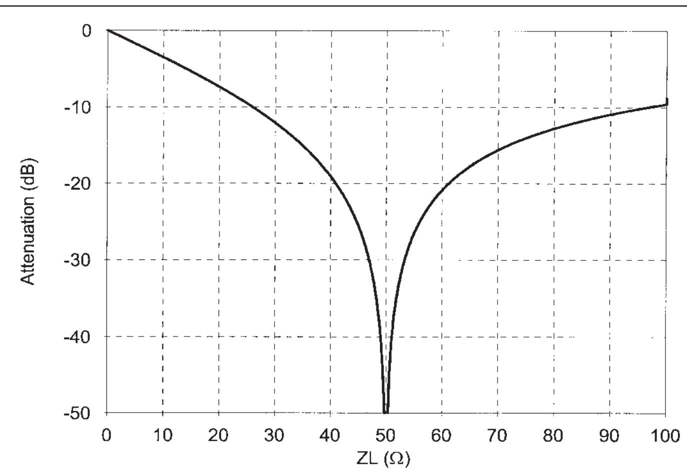


▲ Figure 3. Attenuation values for Z_L from 0 to $2Z_0$.

attenuation vs. Z_L reverses. As Z_L approaches Z_0 the slope becomes very steep making it difficult to set the attenuation accurately beyond approximately 30 dB in direct open loop control. In a closed loop control system, such as an AGC, the sign reversal of the slope could cause instability or gross failure of the control loop unless a slope detection algorithm or other means of detecting and accounting for the sign of the slope is employed.

This problem is eliminated in hardware by the circuit shown in Figure 4. The idea is to place a resistor equal to Z_0 (usually 50 ohms) in shunt with each PIN diode. This guarantees that the termination Z_L on each coupled port will always be less than Z_0. Therefore, only the curve with negative slope on the left side of figure 3 will be selected. Figure 5 shows the attenuation range that we can expect using this kind of arrangement.

From expression (3) in the analysis, we get

$$s_{21}[\text{dB}] = 20\,\text{Log}(|\Gamma_c|) = 20\text{Log}|(Z_L - Z_0)/(Z_L + Z_0)|. \quad (4)$$

Equation 4 can be solved for Z_L as follows. For Z_L and Z_0 positive real and $Z_L < Z_0$

$$Z_L = \frac{Z_0\left(1-10^{\frac{s_{21}}{20}}\right)}{1+10^{\frac{s_{21}}{20}}} \quad (5)$$

where s_{21} is in dB.

For Z_L and Z_0 positive real and $Z_L > Z_0$

$$Z_L = \frac{Z_0\left(1+10^{\frac{s_{21}}{20}}\right)}{1+10^{\frac{s_{21}}{20}}} \quad (6)$$

where s_{21} is in dB.

From (5) we can determine the minimum and maximum terminations required for a desired range of

Step 3:

$$[GG]_{ABCD} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$[GG]_{ABCD} = [AA]_{ABCD}[BB]_{ABCD}[CC]_{ABCD}DD_{ABCD}[EE]_{ABCD}$$

$$[GG]_{ABCD} = \begin{bmatrix} 0 & \frac{jZ_0}{\sqrt{2}} \\ \frac{j\sqrt{2}}{Z_0} & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{R1} & 1 \end{bmatrix}\begin{bmatrix} 0 & jZ_0 \\ \frac{j}{Z_0} & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{R1} & 1 \end{bmatrix}\begin{bmatrix} 0 & \frac{jZ_0}{\sqrt{2}} \\ \frac{j\sqrt{2}}{Z_0} & 0 \end{bmatrix}$$

Step 4:

$$[GG]_Y = \begin{bmatrix} \frac{D}{B} & \frac{(BC-AD)}{B} \\ \frac{-1}{B} & \frac{A}{B} \end{bmatrix}$$

$$[GG]Y = \begin{bmatrix} \frac{2R2}{R1R2+Z_0^2} & \frac{-2jR1R2}{Z_0\left(R1R2+Z_0^2\right)} \\ \frac{-2jR1R2}{Z_0\left(R1R2+Z_0^2\right)} & \frac{2R1}{R1R2+Z_0^2} \end{bmatrix}$$

Step 5:

$$[Y]_{\text{TotalNetwork}} = [FF]_Y + [GG]_Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$$[Y]_{\text{TotalNetwork}} = \begin{bmatrix} 0 & \frac{j}{Z_0} \\ \frac{j}{Z_0} & 0 \end{bmatrix}\begin{bmatrix} \frac{2R2}{R1R2+Z_0^2} & \frac{-2jR1R2}{Z_0\left(R1R2+Z_0^2\right)} \\ \frac{-2jR1R2}{Z_0\left(R1R2+Z_0^2\right)} & \frac{2R1}{R1R2+Z_0^2} \end{bmatrix}$$

$$[Y]_{\text{TotalNetwork}} = \begin{bmatrix} \frac{2R2}{R1R2+Z_0^2} & \frac{j\left(Z_0^2-R1R2\right)}{Z_0\left(R1R2+Z_0^2\right)} \\ \frac{j\left(Z_0^2-R1R2\right)}{Z_0\left(R1R2+Z_0^2\right)} & \frac{2R1}{R1R2+Z_0^2} \end{bmatrix}$$

(continued on page 50)

attenuation. For attenuation ranging from 1 to 40 dB the required termination, Z_L, would range from just under 3 ohms to 49 ohms. In order to achieve 1 dB insertion loss the PIN diodes in Figure 4 would have to have somewhat less than 3 ohms RF resistance to compensate for coupler (and other) losses. Because of the 51 ohm shunt resistor, at the high end of the attenuation scale (40 dB) the diodes would have to present approximately 1250 ohms of RF resistance.

Maximum attenuation of 40 dB would be achievable if parasitic reactances are sufficiently tuned out. Thus, Figure 5 represents the attenuation range (within a few dB) that is reasonably achievable with this configuration. The input and output return losses, ideally infinite, were measured as greater than 19 dB and 16 dB respectively over the attenuation range.

A similar circuit could be constructed that would select the attenuation curve (with positive slope) on the right hand side of Figure 3. In this case, instead of 50 ohm resistors in shunt with the coupled ports we would place the 50 ohm resistors in series with the ports.

Unequal coupled port terminations

From the s-parameter matrix of expression 1, we can see that the effect of unequal coupled port terminations (R1 ≠ R2) is that s_{11} and s_{22} become nonzero making the input port and output ports reflective. The return loss in dB for either the input or output port can be obtained from expression 1 as

$$R_L = -s_{11}[\mathrm{dB}] = -s_{22}[\mathrm{dB}] = -20Log\left|\frac{Z_0(R2-R1)}{(R1+Z_0)(R2+Z_0)}\right| \quad (7)$$

The magnitude of the attenuation produced between input and output ports (labeled Port 1 and Port 2 respectively in Figure 1) is equal to the magnitude of s_{21} in dB. For unequal coupled port terminations, the attenuation can be obtained from expression 1 as

$$\mathrm{Attenuation}[\mathrm{dB}] = -s_{21}[\mathrm{dB}] = -20Log\left|\frac{(R1R2-Z_0^2)}{(R1+Z_0)(R2+Z_0)}\right| \quad (8)$$

As stated above, it is evident

Step 6:

$$[S]_{\mathrm{TotalNetwork}} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}$$

where

$$s_{11} = \frac{(1-y_{11}Z_0)(1+y_{22}Z_0)+y_{12}y_{21}Z_0^2}{(1+y_{11}Z_0)(1+y_{22}Z_0)-y_{12}y_{21}Z_0^2}$$

$$s_{12} = \frac{2y_{12}Z_0}{(1+y_{11}Z_0)(1+y_{22}Z_0)-y_{12}y_{21}Z_0^2}$$

$$s_{21} = \frac{2y_{21}Z_0}{(1+y_{11}Z_0)(1+y_{22}Z_0)-y_{12}y_{21}Z_0^2}$$

$$s_{22} = \frac{(1+y_{11}Z_0)(1-y_{22}Z_0)+y_{12}y_{21}Z_0^2}{(1+y_{11}Z_0)(1+y_{22}Z_0)-y_{12}y_{21}Z_0^2}$$

$$[S]_{\mathrm{TotalNetwork}} = \begin{bmatrix} \frac{Z_0(R1-R2)}{(R1+Z_0)(R2+Z_0)} & \frac{j(R1R2-Z_0^2)}{(R1+Z_0)(R2+Z_0)} \\ \frac{j(R1R2-Z_0^2)}{(R1+Z_0)(R2+Z_0)} & \frac{Z_0(R2-R1)}{(R1+Z_0)(R2+Z_0)} \end{bmatrix} \quad (1)$$

With the s-parameters now in hand, we can quickly assess several operating conditions. Although R1 and R2 were shown schematically as real terminations (pure resistors) in Figures 1 and 2, there has been nothing in the work done so far that prevents them from being complex impedances. Consider the case where R1 and R2 are equal and complex.

$$[S]_{\mathrm{TotalNetwork}} = \begin{bmatrix} 0 & \frac{j(Z_L-Z_0)}{Z_L+Z_0} \\ \frac{j(Z_L-Z_0)}{Z_L+Z_0} & 0 \end{bmatrix} \quad \text{for } R1 = R2 = Z_L \quad (2)$$

$$[S]_{\mathrm{TotalNetwork}} = \begin{bmatrix} 0 & j\Gamma_c \\ j\Gamma_c & 0 \end{bmatrix} \quad (3)$$

where $\Gamma_c = (Z_L - Z_0)/(Z_L + Z_0)$ is the reflection coefficient at the terminated (coupled) ports.

from (7) that the return loss is greatest when the difference between coupled port terminations R1 and R2 is minimal. Conversely, the return loss tends to deteriorate as the difference between R1 and R2 grows. However, this is not necessarily true of attenuation. If R1 = R2 = Z_L is considered the nominal case, then the attenuation is changed minimally if R1 and R2 vary from Z_L equally in opposite directions. The worst case deviation from nominal attenuation occurs when R1 and R2 are equal but vary from Z_L in a direction of steepest curvature (attenuation vs Z_L curve), as shown in Figure 3. Remember, although this is the worst case error for attenuation, it is a best case condition for return loss. This can be illustrated by a specific numerical example:

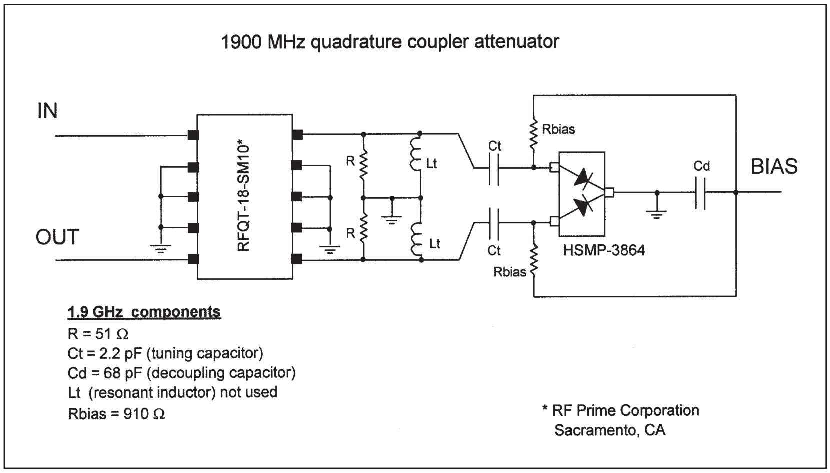
For Z_0 = 50 ohms and a nominal attenuation of 20 dB, R1 = R2 = Z_L where Z_L = 40.91 ohms. According to expression (5),

Case 1 — R1 and R2 vary oppositely by 5%: R1 = 0.95 Z_L, R2 = 1.05 Z_L.

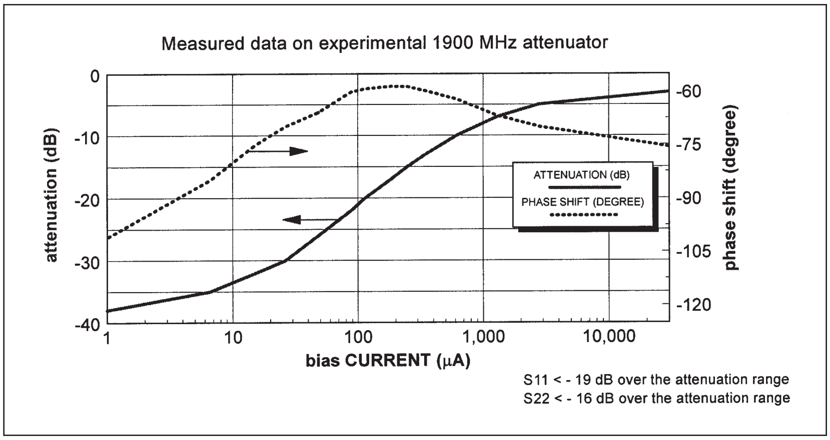
Attenuation = 19.95 dB
Attenuation error = .05 dB
Return Loss = 32.12 dB

Case 2 — R1 and R2 vary oppositely by 20%: R1=0.8 Z_L, R2=1.2 Z_L.

Attenuation = 19.25 dB
Attenuation error = .75 dB
Return Loss = 20 dB



▲ **Figure 4. Circuit of a practical coupler-based attenuator. (Courtesy of Hewlett-Packard Company)**



▲ **Figure 5. Attenuation range expected from the circuit of Figure 5. (Courtesy of Hewlett-Packard Company)**

Case 3 — R1 and R2 vary equally by +20%; R1 = R2 = 1.2 Z_L.

Attenuation = 40.75 dB
Attenuation error = 20.75 dB
Return Loss = infinite

Case 4 — R1 and R2 vary equally by –20%; R1 = R2 = 0.8 Z_L.

Attenuation = 13.61 dB
Attenuation error = 6.39 dB
Return Loss = infinite

Case 1 and 2 demonstrate that although the return loss degrades with large (opposite) variations in coupled port terminations, the attenuation error is very small (0.75 dB for ±20% variation from Z_L). One reason for this is that as one termination is more reflective than the nominal value, the other is less reflective and tends to compensate. Of course this kind of variation will not be the usual case. However, it does serve to illustrate that if terminations R1 and R2 are unequal, the resulting attenuation will be approximately equal to the value the attenuator would produce if both ports were terminated in the mean value of R1 and R2 (particularly the geometric mean).

Cases 3 and 4 represent the ideal attenuator with equal terminations offset from the nominal value. This simply moves the point on the attenuation curve of Figure 3 either side of the nominal value. This is similar to the case where the terminations are provided by two well matched, co-packaged PIN diodes as shown in Figure 4. The dice for the Hewlett-Packard HSMP-3864 PIN diode pair are cut from adjacent sites on the same wafer. As a result, if the internal and external parasitics are tuned out around the HSMP-3864, a device like this will provide excellent tracking between each coupled port for all bias currents [3]. The "offset" error can then be corrected by adjusting the bias current.

Conclusions

A method was presented for the analysis of a variable attenuator formed from a 3 dB quadrature coupler with variable tracking coupled port terminations. The analysis used simple two-port methods to derive the *s*-parameter matrix for the attenuator with arbitrary terminations. The *s*-parameters were then used to arrive at general and specific results, detailing the performance of the attenuator under conditions of unequal and equal coupled port terminations.

The results indicate that this variable attenuator has the capability to provide excellent return loss at both the input and output ports over a wide range of attenuation. Additionally, it has the advantage of simplified attenuation control circuitry.

Acknowledgments

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References

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