An Efficient Procedure for Narrowband Bandpass Filter Design

Here is a review of lowpass-to-bandpass transformations with an example filter operating at 5 GHz

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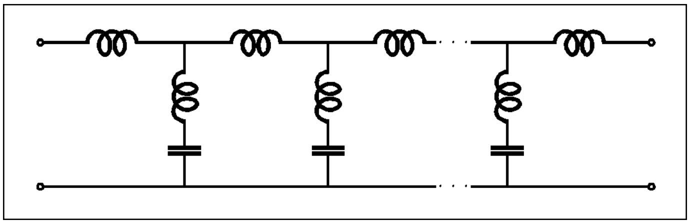
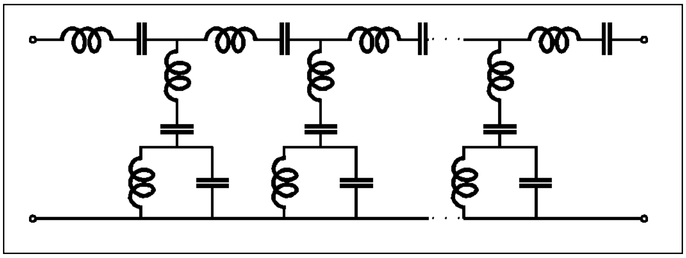
Bandpass filters (BPF) may be constructed by cascading lowpass (LP) and highpass (HP) filters. This type of realization is suitable for devices with wide bandwidths but is not convenient for the realization of narrowband BPFs, that is, for bandwidths of less than about 10 percent [1]. This is related to the high degree (number of poles and zeros) needed for the filters to achieve good selectivity. The cascaded type of realization causes large insertion losses and poor amplitude flatness.



▲ **Figure 1. LP prototype filter with real transmission zeros.**



▲ **Figure 2. BPF structure.**

Cauer and generalized Chebyshev [2, 3] LP prototype filters are very good starting points for the design of selective BPFs. This article discusses the BPF network transformations suitable for physical realization [4, 5]. These transformations are based on the insertion of redundancy ideal transformers in BPFs and the use of Norton's equivalent networks [6].

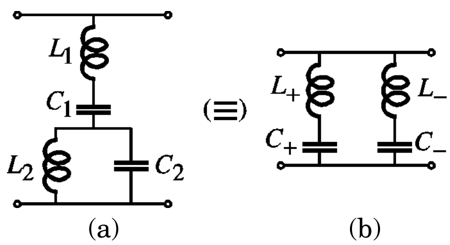
A proper choice of transformer transformation ratio produces a network with a minimum spread of element values. A new and efficient numerically-based procedure for getting an exact solution of optimal parameter t in closed form is presented in [5]. The lumped element network ultimately obtained has no transformers and is very convenient for the design of microwave filters, diplexers and multiplexers using printed circuit technology.

This article also discusses the efficient transformation for the design of a narrowband bandpass filter with transmission lines. The use of suspended substrate techniques [1, 7] allows us to use highly selective prototypes, which can achieve excellent performances. The suspended substrate stripline (SSS) narrowband BPF design procedure is introduced by an example filter with excellent characteristics.

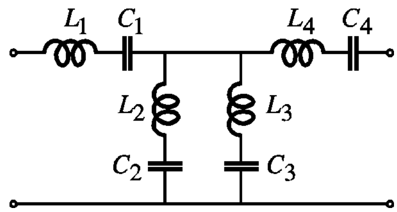
Lumped element network development

A typical LP prototype filter with real transmission zeros is shown in Figure 1. The BPF in Figure 2 is obtained from the use of a well-known frequency transformation on element immitances of the LP prototype filter.

The transformed network in Figure 2 has a large spread of element values and shunt structures unsuitable for practical construction. Since these are very difficult to realize directly, a more convenient equivalent circuit must be found. Such an equivalent structure has been derived [1] and is shown in Figure 3b.



▲ **Figure 3. Alternative shunt element configurations.**



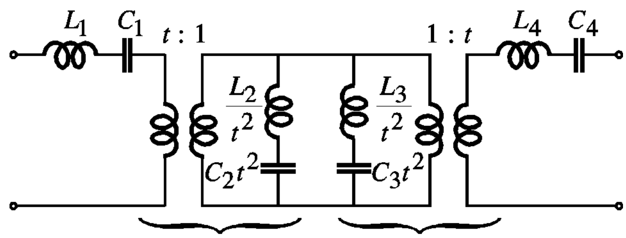
▲ **Figure 4. Transformed BPF.**

The element values L_+, C_+, L_-, C_- of the network in Figure 3b, are defined by relations [1]

$$
\begin{aligned}
a &= L_1C_1 + L_2C_2 + L_2C_1 \\
b &= L_1L_2C_1C_2 \\
\alpha_+ &= \sqrt{\frac{a}{2} + \left(\frac{a}{2}\right)^2 - b} \\
\alpha_- &= \frac{a}{2} + \sqrt{\left(\frac{a}{2}\right)^2 - b} \\
\beta_+ &= \frac{C_1L_2C_2 - C_1\alpha_+}{\alpha_- - \alpha_+} \\
\beta_- &= \frac{C_1L_2C_2 - C_1\alpha_-}{\alpha_+ - \alpha_-} \\
C_+ &= \beta_+, C_- = \beta_-, \\
L_+ &= \frac{\alpha_+}{\beta_+} \text{ and } L_- = \frac{\alpha_-}{\beta_-}
\end{aligned}
\tag{1}
$$

By transforming the network in Figure 2 into that of Figure 3, a more realizable network can be obtained. The spread of element values is smaller than in the original BPF, but it is still quite large. The simplest third degree network obtained in this manner is shown in Figure 4. This structure has the additional disadvantage that branches connected in parallel cause unwanted parasitic capacitance and inductance [6].

The parasitic effects can be removed and the spread of element values decreased by insertion of redundant ideal transformers with the transformation ratio t. For the network in Figure 4, two ideal transformers can be inserted as in Figure 5. As we can see, both networks have the same transfer function. Ideal transformers can be eliminated by Norton's equivalent networks [6] given in Figure 6. Equivalencies should be used twice on the parts of the network in Figure 5 designated with large brackets. It should be noted that one of the serial impedances in Norton's equivalent network always has a negative value. In many instances it is possible, using the other part of the network, for the ultimately transformed network to have all positive element values. The network in Figure 7 is obtained by transforming the network in Figure 5 with equivalencies in Figure 6. The last BPF element values are



▲ **Figure 5. BPF with ideal transformers.**

$$
\begin{aligned}
L_1' &= L_1 + L_2\frac{t-1}{t}, \frac{1}{C_1'} = \frac{1}{C_1} + \frac{t-1}{C_2t} \\
L_2' &= \frac{L_2}{t}, C_2' = C_2t \\
L_3' &= (L_2 + L_3)\frac{1-t}{t^2}, \frac{1}{C_3'} = \left(\frac{1}{C_2} + \frac{1}{C_3}\right)\frac{1-t}{t^2} \\
L_4' &= \frac{L_3}{t}, C_4' = C_3t \\
L_5' &= L_4 + L_3\frac{t-1}{t}, \frac{1}{C_5'} = \frac{1}{C_4} + \frac{t-1}{C_3t}
\end{aligned}
\tag{2}
$$

A sufficient condition for all the network element values in Figure 7 to be positive is

$$
\max\left\{\frac{L_2}{L_1+L_2}, \frac{C_1}{C_1+C_2}, \frac{L_3}{L_3+L_4}, \frac{C_4}{C_3+C_4}\right\} < t < 1 \tag{3}
$$

By inserting the two ideal transformers in the network shown in Figure 5, the resulting network has two additional elements. Element values depend on the value of the redundancy factor t, and the last network is without transformers. By inserting two transformers with different transformation ratios t_1 and t_2, it may be possible for the transformed network to have only one additional element. In this case the transformed network is also without transformers, but has different loads, $R_S \neq R_L$.

The optimum value of parameter t can be found in closed form [5]. The first step in getting optimal t is calculating maximum and minimum values of inductances and capacitances for the discrete values of the parameter t from the range defined by Equation (3). If the aim of the BPF design is to minimize the ratio between the maximum and minimum values of reactive elements, it can be noted that for the chosen step Δt, there is a value t for which this condition is fullfilled. The next step is equating minimum or maximum values of inductances or capacitances that change values in the vicinity of the chosen value t. This gives an equation which yields the optimum value of t.

Distributed element network development

The next step in designing a narrowband BPF is to transform the lumped element BPF into one with distributed elements. Inductors are replaced with short-circuited capacitors and capacitors with open-circuited transmission lines. In the case of the third degree BPF given in Figure 7, a transformed BPF with ideal transmission lines is given in Figure 8. The electrical length of all transmission lines is $\theta_c = 45°$ at the band center frequency f_c.

The BPF obtained this way usually has transmission lines with characteristic impedance values that are too high to be practical for realizaton in printed circuit technology. Therefore, a transformation should be applied on the last BPF. The impedance values of the filter could be scaled to the lower values:

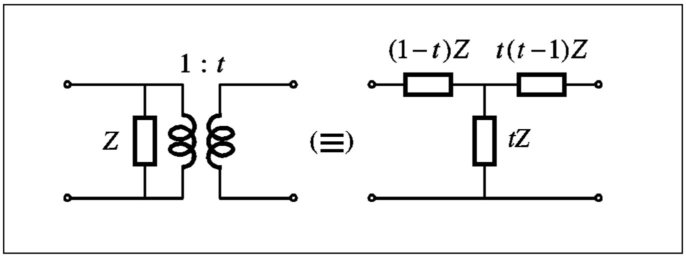
$$Z'_{ci} = \frac{Z_{ci}}{n},\ i = 1,2,\ldots,10 \tag{4}$$

and gyrators should be introduced at input and output to transform all impedances to the characteristic impedance at the input and output port, Z_c. The gyrators' constant is:

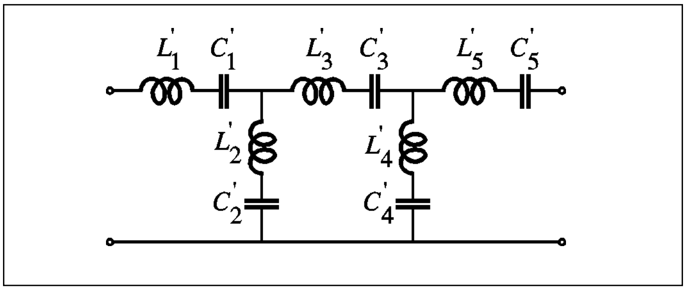
$$r = \frac{Z_c}{\sqrt{n}} \tag{5}$$

Parameter n from the equations (4) and (5) is a positive number.

A ladder BPF network can be efficiently characterized by ABCD parameters, as seen in Figure 9. For reciprocal networks $AD - BC = 1$, and for symmetrical networks $A = D$. The use of ABCD parameters at microwave frequencies is not very convenient from the measurement point of view. Scattering matrix formulation is a more general method of representing microwave networks. Therefore, conversion from the ABCD-matrix to the S-matrix for reciprocal and symmetrical network from Figure 9, gives [8]:



▲ **Figure 6. Norton's equivalent networks.**

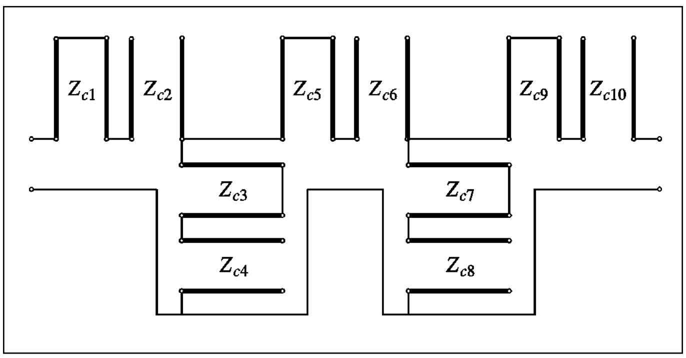


▲ **Figure 7. BPF structure ultimately obtained.**

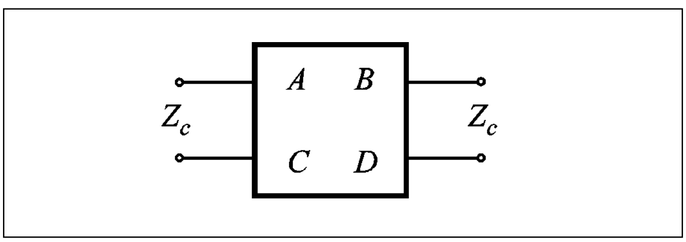
$$S_{11} = S_{22} = \frac{B - CZ_c^2}{2AZ_c + B + CZ_c^2} \tag{6}$$

$$S_{12} = S_{21} = \frac{2Z_c}{2AZ_c + B + CZ_c^2} \tag{7}$$

A transformed network from Figure 9 with scaled impedance values as in (4). This network with two gyrators defined by (5) is shown in Figure 10. It can be shown that this network has the next S-matrix



▲ **Figure 8. The BPF with ideal transmission lines.**



▲ **Figure 9. A generalized two-port network.**

$$S_{11}^g = S_{22}^g = -S_{11} = -S_{22} \tag{8}$$

$$S_{12}^g = S_{21}^g = S_{12} = S_{21} \tag{9}$$

Equations (8) and (9) show that the transformed network has the same S-parameters as the original network. The only difference is a phase shift of 180° for the reflection coefficients S_{11}^g and S_{22}^g. This fact enables the use of this network for the BPF realization. In the case of a narrowband BPF it is possible to realize gyrators as low impedance transmission lines one-quarter wavelength long at band center, with characteristic impedance $Z'_c = r$ and electrical length $\theta = 90°$ at the center frequency f_c.

In the next step, series connected, short-circuited and open-circuited transmission lines in parallel branches should be replaced with cascade-connected transmission lines to obtain shunt stubs suitable for practical realization. This is illustrated in Figure 11 and characteristic impedance values are defined by

$$Z_{c3} = Z_{c1} + Z_{c2}, Z_{c4} = \frac{Z_{c1}}{Z_{c2}} Z_{c3} \tag{10}$$

Another problem for printed circuit realization of the BPF is existance of short-circuited transmission lines in series branches. In the case of the third degree filter, those are scaled values Z'_{c1}, Z'_{c5} and Z'_{c9}. This disadvantage can be removed by realizing these lines using short transmission lines with high characteristic impedance values Z_{ch} and electrical lengths:

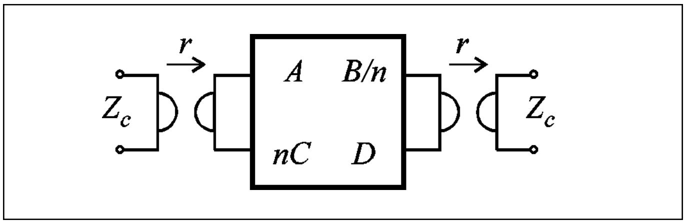
$$\theta_i = \frac{Z'_{ci}}{Z_{ch}} \frac{180}{\pi} [°], i = 1,5,9 \tag{11}$$

The final BPF structure with ideal transmission lines is given in Figure 12.

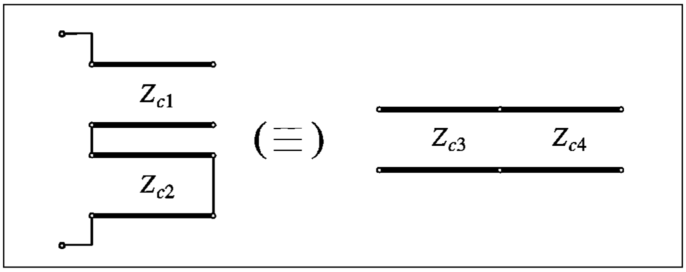
BPF design example

A design procedure for BPF in SSS technology can be given by example. A narrowband BPF, with passband frequencies of f_1 = 4875 MHz and f_2 = 5125 MHz, is a particularly good design example. In this case, the center frequency is $f_c = (f_1 f_2)^{1/2}$ = 4998.4373 MHz and the bandwidth is approximately 5 percent of f_c. Cauer's LP prototype third degree filter given in Figure 13 is transformed into a bandpass prototype filter (BPPF) with the normalized center frequency $\Omega_{cn} = 1$. The LP prototype filter C0325-28 [9] has maximum attenuation in the pass-band α_p = –0.2803 dB, minimum attenuation in the stop-band α_s = –30.4 dB, and element values are L_{1p} = 1.221240, C_{2p} = 0.985344 and L_{2p} = 0.172857. From the Richards frequency transformation

$$\Omega_{cn} = \tan\theta_c = \tan\omega_c T \tag{12}$$



▲ **Figure 10. A transformed two-port network.**



▲ **Figure 11. Equivalent circuits.**

the time delay $T = 1/(8 f_c) = 2.5008 \times 10^{-11}$ s can be calculated. The normalized bandwidth of the BPPF is $B_n = \tan\omega_2 T - \tan\omega_1 T$ = 0.0786449. Applying the previously presented procedure, the LP prototype filter given in Figure 13 can be transformed into a BPPF as shown in Figure 7.

The optimum value of parameter t can be found in closed form [5]. Equation (3) gives $0.2382307<t<1$. The first step in getting an optimum value of t is calculating maximum and minimum values of inductances and capacitances for the discrete values of the parameter t. It is shown in Table 1.

If the aim of the BPF design is to minimize the ratio between the maximum and minimum values of reactive elements, it can be seen from Table 1 that for $0.3 \le t \le 0.9$ and step Δt = 0.1, t = 0.6. Maximum and minimum values of reactive elements change the values in the vicinity of t = 0.6. Therefore, it is necessary to calculate extreme values for t = 0.55 and t = 0.65.

For t = 0.55, the values are

$L_{max} = L'_3 = 13.1974$, $L_{min} = L'_2 = 7.3001$,
$C_{max} = C'_4 = 0.136984$, $C_{min} = C'_3 = 0.0757725$,
$L_{max}/L_{min} = C_{max}/C_{min} = 1.808$

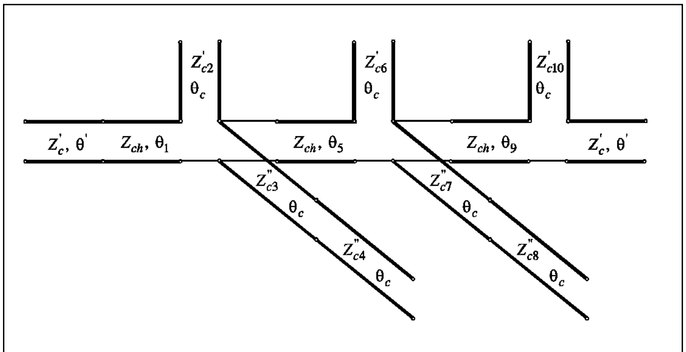
For t = 0.65, the values are

$L_{max} = L'_1 = 13.3666$, $L_{min} = L'_2 = 6.17701$,
$C_{max} = C'_4 = 0.161891$, $C_{min} = C'_5 = 0.0748134$,
$L_{max}/L_{min} = C_{max}/C_{min} = 2.164$

To obtain a closed form for the exact values of optimal parameter t, it is necessary to notice that for $t < 0.55$ the maximum inductance value is $L_{max} = L'_3$, and for $t >$ 0.55 it is $L_{max} = L'_1$. Equating $L'_3 = L'_1$, the optimal

t	L_{max}	L_{min}	L_{max}/L_{min}	C_{max}	C_{min}	C_{max}/C_{min}
0.3	L'_3 = 69.0012	L'_5 = 4.19663	16.442	C'_1 = 0.238287	C'_3 = 0.0144925	16.442
0.4	L'_3 = 33.2684	L'_5 = 8.24374	4.035	C'_1 = 0.121304	C'_3 = 0.0300585	4.035
0.5	L'_3 = 17.7432	L'_2 = 8.03011	2.209	C'_4 = 0.124531	C'_3 = 0.0563597	2.209
0.6	L'_1 = 12.8518	L'_2 = 6.69176	1.920	C'_4 = 0.149438	C'_5 = 0.0778099	1.920
0.7	L'_1 = 13.8078	L'_3 = 5.43158	2.542	C'_3 = 0.184108	C'_5 = 0.0724229	2.542
0.8	L'_1 = 14.5248	L'_3 = 2.77237	5.239	C_3 = 0.360702	C'_5 = 0.0688479	5.239
0.9	L'_1 = 15.0824	L'_3 = 1.09526	13.771	C'_3 = 0.913027	C'_5 = 0.0663024	13.771

▲ **Table 1. The extreme element values for 0.3 ≤ t ≤ 0.9.**



▲ **Figure 12. The approximated BPF structure with ideal transmission lines.**

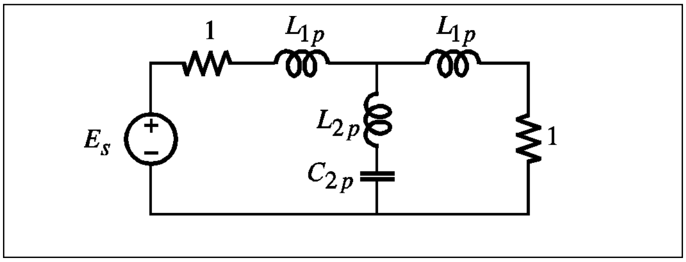
redundancy parameter

$$t = -\frac{L_3}{2(L_1+L_2)} + \sqrt{\left[\frac{L_3}{2(L_1+L_2)}\right]^2 + \frac{L_2+L_3}{L_1+L_2}} \qquad (13)$$

is obtained and the calculated value is t = 0.560862. The same value could be calculated from the requirement $C'_5 = C'_3$, because we started from the symmetrical LP prototype filter. The element values for the BPPF are L'_1 = 12.3849, C'_1 = 0.0852805, L'_2 = 7.15873, C'_2 = 0.115486, L'_3 = 12.3849, C'_3 = 0.0807437, L'_4 = 8.65905, and C'_5 = 0.0807437. The ratio between the maximum and minimum values is $L_{max}/L_{min} = C_{max}/C_{min}$ = 1.73. The obtained structure of the BPPF is very convenient for realization in the printed circuit technology, because it has a minimum spread of element values.

The next step in SSS BPF designing is transforming the BPPF into the BPF with ideal transmission lines. The characteristic impedance at the input and output port is Z_c = 50 Ω. This BPF is shown in Figure 8. All transmission lines have the same electrical lengths of θ_c = 45° at the center frequency f_c. The characteristic impedance values are

$Z_{c1} = L'_1 Z_c$ = 619.245 Ω, $Z_{c2} = Z_c/C'_1$ = 586.3005 Ω,
$Z_{c3} = L'_2 Z_c$ = 357.9365 Ω, $Z_{c4} = Z_c/C'_2$ = 432.9525 Ω,
$Z_{c5} = L'_3 Z_c$ = 619.245 Ω, $Z_{c6} = Z_c/C'_3$ = 619.24336 Ω,



▲ **Figure 13. Double terminated LP prototype filter.**

$Z_{c7} = L'_4 Z_c$ = 432.9525 Ω, Z_{c8} = 357.9543 Ω,
$Z_{c9} = L'_5 Z_c$ = 586.3 Ω, and
$Z_{c10} = Z_c/C'_5$ = 619.24336 Ω

It can be seen that transmission lines have too high characteristic impedance values to be practical in SSS technology. Therefore, a transformation should be applied on the last BPF by scaling impedance values and introducing gyrators as described earlier. For this example, n = 16 in equations (4) and (5), and the electrical length of the gyrators is θ = 90° at the center frequency f_c.

Shunt stubs suitable for practical realization are obtained by using the transformation given in Figure 11 and equation (10).

Short-circuited transmission lines in series branches may be realized using short transmission lines with high characteristic impedance, Z_{ch} = 150 Ω, and electrical lengths given in Equation (11).

The final BPF structure with ideal transmission lines is shown in Figure 12. The element values are

Z'_c = 12.5 Ω, Z_{ch} = 150 Ω, Z'_{c2} = 36.643781 Ω,
Z''_{c3} = 49.430588 Ω, Z''_{c4} = 59.790262 Ω,
Z'_{c6} = 38.70271 Ω, Z''_{c7} = 49.430495 Ω,
Z''_{c8} = 40.865742 Ω, Z'_{c10} = 38.70271 Ω,
θ' = 90°, θ_1 = 14.783386°, θ_c = 45°,
θ_5 = 14.783386°, θ_9 = 13.996881°

All electrical lengths are calculated at $f = f_c$.

The next step in the BPF design is the calculation of real printed circuit parameters with SSS and broadside coupled SSS (BCSSS) transmission lines. The chosen

	Z'_c θ	Z_{ch} θ_1	Z'_{c2} θ_c	Z''_{c3} θ_c	Z''_{c4} θ_c	Z_{ch} θ_5	Z'_{c6} θ_c	Z''_{c7} θ_c	Z''_{c8} θ_c	Z_{ch} θ_9	Z'_{c10} θ_c
w (mm)	27.11	0.58	3.087	5.39	4.09	0.58	5.706	5.39	6.99	0.58	8.889
l (mm)	14.51	2.15	4.542	6.97	6.91	2.15	3.355	6.97	7.01	2.03	2.361

▲ **Table 2. Physical dimensions of SSS and BCSSS transmission lines for BPF.**

substrate is Cu-Flon with relative dielectric constant $\varepsilon_r = 2.1$, substrate thickness $h = 0.7874$ mm, conductor thickness $t = 0.0127$ and upper and lower ground plane to substrate spacing $h_u = h_l = 1.7$ mm. All transmission lines except Z'_{c2}, Z'_{c6}, and Z'_{c10} are synthesized by using SSS transmission lines in the LineCalc program. Open-circuited transmission lines with the characteristic impedances Z'_{c2}, Z'_{c6}, and Z'_{c10} are modeled with BCSSS transmission lines [10] by use of the tuning and optimization procedure in the Libra program. The calculated width and length values of all transmission lines are given in the Table 2.

The last step is drawing the BPF layout in the GasStation program. Calculated transmission characteristics of the designed filter are shown in Figure 14. The given characteristics are for the filter in Figure 8 (BPFIT), for the filter in Figure 12 (BPFITA) and for the filter in suspended substrate technology (BPFSSS). It can be seen that the designed filter in SSS technology corresponds very well to the filters with ideal transmission lines. The layout for designed filter is shown in Figure 15. The designed filter should be realized and tested very soon and measured characteristics will be given in a future article.

Conclusion

This article has presented A design technique for a physically realizable selective band-pass filter. This technique enables narrow-band SSS filters to be designed. In this article, transformations of BPFs have been given based on the insertion of ideal transformers and the use of Norton's equivalencies. The closed form of the optimal redundancy parameter t has been obtained. A few very efficient transformations and approximations convenient for the design of BPF with transmission lines have been given. The design procedure has introduced by one example of BPF designed in SSS and BCSSS technology. It is shown that a BPF can be designed very efficiently by using the presented technique. ■

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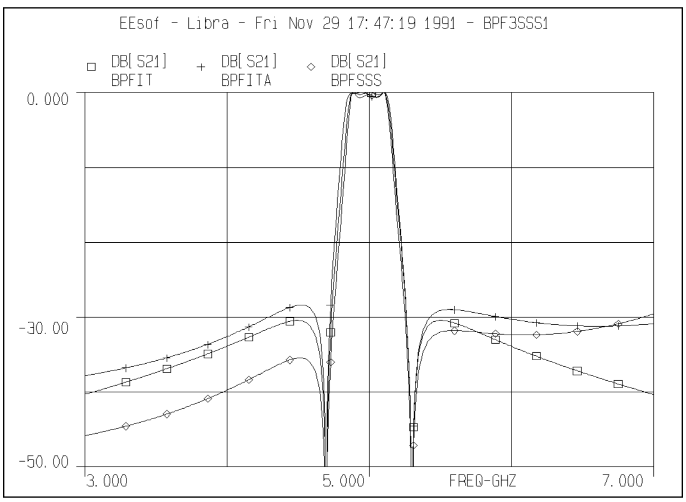
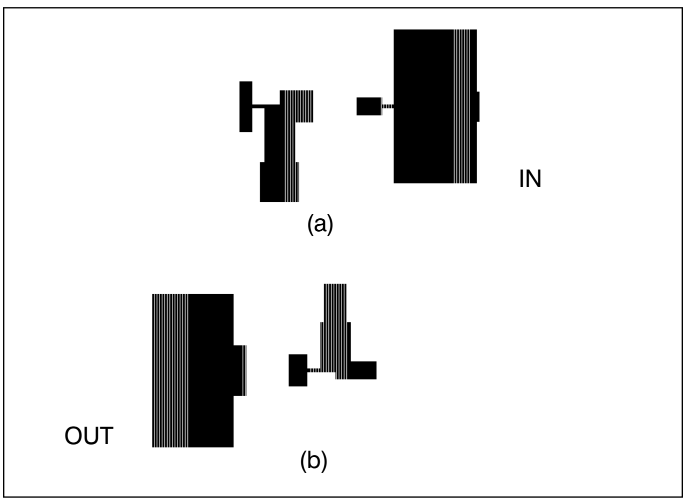


▲ **Figure 14. Transmission characteristics of the designed filters.**



▲ **Figure 15. BPF layout a) upper side and b) lower side.**

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