RF Capacitive-Coupled Filters

Here is a practical review of a common bandpass filter topology — one of the simplest types to implement in production
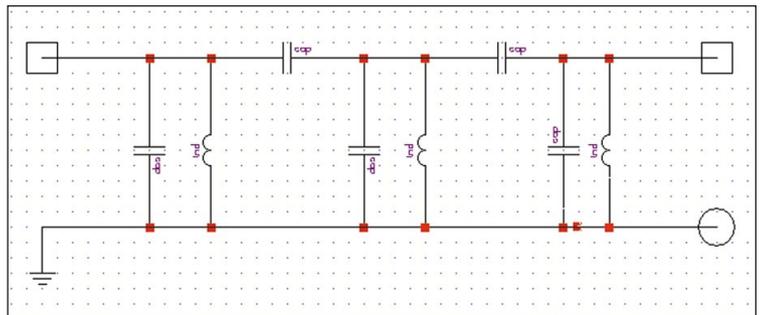
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One of the more practical filters available for RF narrow-band applications is the capacitive coupled filter. Particular variations of the basic configuration can also be quite useful. For example, some manipulation with impedance matching can provide better real-world component values. Also, higher or lower out-of-band roll-off can be implemented in the design. Another special feature is that transmission notches can be inserted for particularly steep roll-

■ Figure 1. General configuration of a capacitive coupled filter.

off. These advantages make the capacitive coupled filter, along with the proposed variations, very useful in a number of applications.

The concepts presented here will show the basic generation of the filter and then will progress to variations that add more capability. This will be done with a combination of hand analysis and C programming for synthesis of the filter. Then analysis and final optimization will be done with Ansoft's Serenade software package (formerly Compact), a powerful Integrated Development Environment (IDE) tool.

The C++ design program determines component values for the configuration of Figure 2. This configuration is generally preferred over others (such as Figure 3c), because it is heavy in capacitors and light in inductors. (Generally, radiation concerns, cost and convenience side with capacitors more than inductors in production.)

The program calls for the center frequency, bandwidth, pass-band ripple, input and output impedance, and allows the shunt inductor

values to be chosen. It then determines the number of stages needed, along with the capacitor design values. If the design values are not practical or desired, the program can be rerun with different inductor values or other variables. The program output also gives Q requirements that the inductors must meet.

The hand analysis presented here allows for adapting to other configurations, such as Figure 3c. Although perhaps not quite as production-oriented, this inductive matching (rather than capacitive) causes a sharper high-end frequency roll-off if desired. In general, capacitive coupled filters give a sharp roll-off on the low frequency
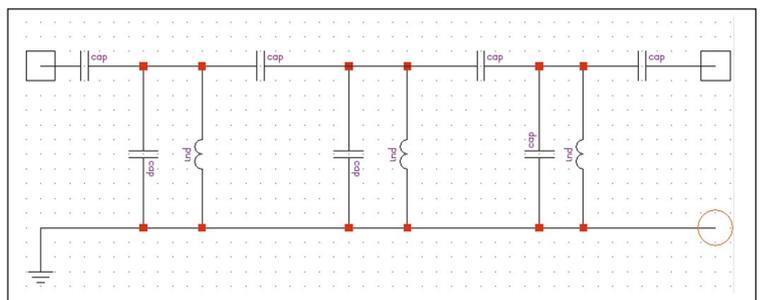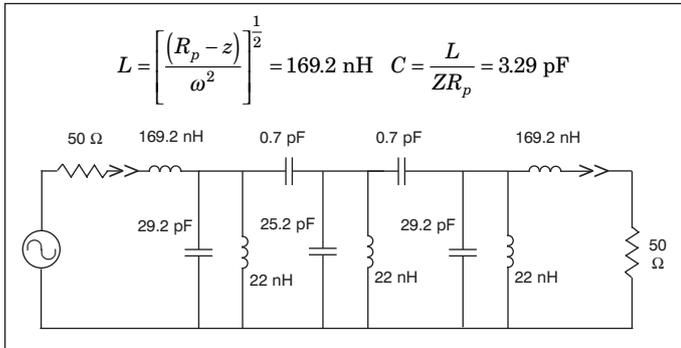

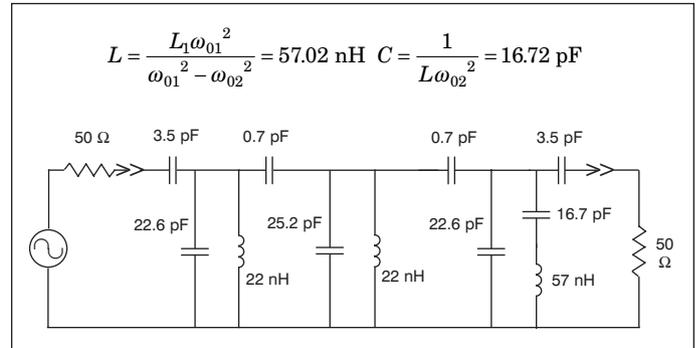■ Figure 2. Standard configuration of a capacitive coupled filter.
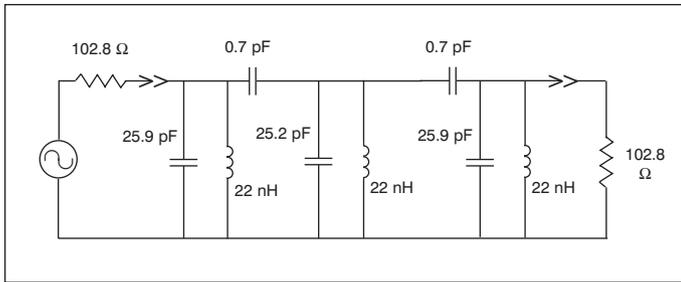
■ **Figure 3a. Example in preliminary form.**
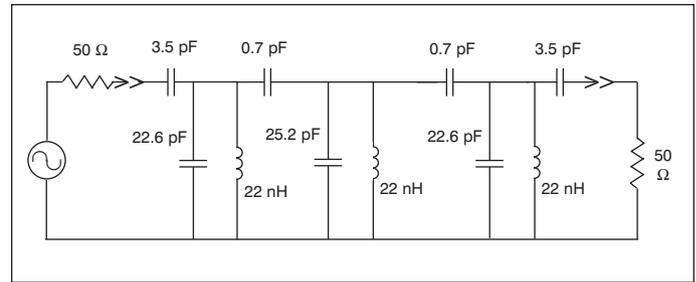
■ **Figure 3b. Example capacitive coupled in 50 ohm form.**

$$L = \left[\frac{(R_p - z)}{\omega^2}\right]^{\frac{1}{2}} = 169.2 \text{ nH} \quad C = \frac{L}{ZR_p} = 3.29 \text{ pF}$$

■ **Figure 3c. Example inductive coupled in 50 ohm form.**

$$L = \frac{L_1\omega_{01}^2}{\omega_{01}^2 - \omega_{02}^2} = 57.02 \text{ nH} \quad C = \frac{1}{L\omega_{02}^2} = 16.72 \text{ pF}$$

■ **Figure 3d. Example L/C resonator in 50 ohm form.**

end but not near as much attenuation on the higher side of the pass-band. This, of course, is due to the series capacitors having a low impedance at the higher frequencies. However, transposing the output series capacitance with an inductor can give the same pass-band characteristics while providing better out-of-band attenuation on the high end.

Figure 3d can also be implemented, which can give an arbitrary notch or a much sharper roll-off on the low frequency side of the pass-band. This basically converts one or more of the shunt inductors to an L/C series resonator. The resonator, at the pass-band frequency, looks like the same pure positive reactance of the inductor it replaced. But at the resonator's resonant frequency, its impedance will be an RF short, effectively grounding or notching the signal out at that frequency.

Ansoft's Serenade package completes the design, with analysis and optimization. Serenade provides the ability to analyze real world Q value affects of the inductors (and caps if needed). It also allows micro-strip modeling, and other effects of the board and component layout. Actual implementation, of course, means using component values that are usually slightly different from the design values. The actual component values and distributed microstrip can be optimized with Serenade for s-parameter impedance and transmission goals. This also includes group delay, an important consideration in many filters. Breadboard testing of the final models has shown excellent correlation of measured results in a broad range of frequencies and applications.

Figure 1 shows the general form of a capacitive coupled filter. Authors such as Zverev [1] and Williams and Taylor [2] present tables with q and k coefficient values used with this configuration. In this form, however, the parameters used with the equations show that there is a trade-off of the inductor values with the input and output characteristic impedance. In other words, impedance or inductor values can be chosen, but not both.

A basic derivation leads to equations (1) and (2) (Appendix I) and Figure 2. In this configuration, one can choose a convenient inductance value as well as both input and output impedances. (Note: The inductor values are all the same, and the input impedance can be different than the output.) In general, practical inductor values force the impedance of configuration 1 to be higher than normally desired. Converting from Figure 1 to Figure 2 can correct this problem.

The derivation shows this transformation is done by adding negative capacitance to the two outside shunt capacitors and then adding positive series shunt capacitance on the output ports. The series capacitances are usually small but real-world values. The negative shunt capacitance is accomplished by subtracting from the already existing positive capacitance generated in Figure 1. In this configuration, the inductors do not change, but the two outside shunt capacitor values do. Thus, the mathematical derivation is physically realizable and actually works very well.

In the following example, the design progression is shown going from Figure 1 to Figure 2 in detailed steps. It should be noted that the chebyshev k and q values used are ripple pass-band coefficients, rather than –3 dB points. In other words, the 208 MHz center frequency will have 0.1 dB 6 MHz pass-band ripple and will imme-

diately begin rolling off outside the pass-band ripple. -3 dB coefficients could also be used if desired [1, 2] in chebyshev, butterworth, or other filter type characteristics. Note that derivations for C_s and C_p used in the design are found in Appendix I. Reference [3] shows the generation of k and q coefficients from g values. The derivation, in turn, of g values may be found in reference [4].

208 MHz capacitive coupled BPF design example

F_0 = 208 MHz, BW = 6 MHz passband
0.1 dB Chebyshev, 3 stage

$$\frac{q_1}{1.0316} \quad \frac{q_3}{1.0316} \quad \frac{k_{12}}{.9192} \quad \frac{k_{23}}{.9192}$$

$$Q_{bp} = \frac{208 \text{ MHz}}{6 \text{ MHz}} = 34.667$$

$$Q_1 = Q_{bp}q_1 = 34.667(1.0316) = 35.762$$

$$Q_3 = Q_{bp}q_3 = 34.667(1.0316) = 35.762$$

$$K_{12} = \frac{K_{12}}{Q_{bp}} = \frac{.9192}{34.667} = 0.0265$$

$$K_{23} = \frac{K_{23}}{Q_{bp}} = \frac{.9192}{34.667} = 0.0265$$

choose L = 22 nH

$$R_S = \omega_0 L Q_1 = 1028.23 \, \Omega$$

$$R_L = \omega_0 L Q_3 = 1028.23 \, \Omega$$

$$C_{node} = \frac{1}{\omega_0^2 L} = 26.6 \text{ pF}$$

$$C_{12} = K_{12}C_{node} = 0.0265(26.6 \text{ pF}) = 0.705 \text{ pF}$$

$$C_{23} = K_{23}C_{node} = 0.0265(26.6 \text{ pF}) = 0.705 \text{ pF}$$

$$C_1 = C_{node} - C_{12} = 26.6 \text{ pF} - 0.705 \text{ pF} = 25.9 \text{ pF}$$

$$C_2 = C_{node} - C_{12} - C_{13} = (26.6 - 0.705 - 0.705)\text{pF}$$
$$= 25.2 \text{ pF}$$

$$C_3 = C_{node} - C_{13} = 26.6 \text{ pF} - 0.705 \text{ pF} = 25.9 \text{ pF}$$

Figure 3a shows the example filter in preliminary form, which is then converted to 50 ohms (Figure 3b) by:

$$C_3 = \left[\frac{1}{w^2 z (R_p - z)}\right]^{\frac{1}{2}} = 3.5 \text{ pF}$$

$$C_p = \frac{C_3(Z - R_p)}{R_p} = -3.3 \text{ pF}$$

Because of the versatility of Figure 2, design synthesis of this configuration was incorporated in C++ programming (Appendix IV) called "capcpl." In this program, the pass-band chevyshev coefficients are generated and the design steps are automatically performed as outlined in the 208 MHz example above. Using the same filter design goals from the 208 MHz example will generate the following program listing:

CPLCAP program 208 MHz design synthesis example

Enter Center Frequency (Hz) fo:208000000
Enter Bandwidth (Hz) BW:6000000
Enter Passband Ripple (dB):.1
Enter Number of Resonator Sections n:3
Enter Inductance L (nH):22
Enter Source Impedance (Ohms):50
Enter Load Impedance (Ohms):50
Q= 35.761623
L= 2.200000e–08
C[1]= 2.261562e–11
C[2]= 2.520159e–11
C[3]= 2.261562e–11
C[0,1]= 3.459832e–12
C[1,2]= 7.056166e–13
C[2,3]= 7.056166e–13
C[3,4]= 3.459832e–12

press any key to exit...

Note the design values match the long-hand detailed example previously given. Also note the general filter quality Q requirement is 35.7. This is usually set by the inductors, which generally have far lower Q values than the capacitors. Not having high Q value components means having more rounding on the pass-band corners and higher pass-band insertion loss. Also the pass-band ripple gets "smoothed out." The effects become more pronounced at smaller bandwidths. At UHF frequencies, practical inductors can provide Q values in the range of 80 and above, which should allow the actual filter to reasonably match the design goals.

Analysis of the 208 MHz design is shown in Figures 4a through 4d. Note how closely the pass-band matches the designed goal of 6 MHz. Also note the reasonable impedance matching shown in the polar and rectangular plots. Finally, note the affects of inductive Qs on the circuit performance S_{21} pass-band gain.

Some general observations are noted for Chebyshev configurations. Normal Chebyshev configurations (L/C in both series and shunt path) require the load impedance to be higher than the source for even-ordered filters. This is not true with the capacitive-coupled case. Even-ordered sections may be designed with equal source and load impedances, the same as for odd-orders.

The pass-band ripple for the even-ordered case will have its lowest "dip" in the center frequency, whereas there will be a "peak" in the center ripple for odd-ordered filters. A related observation (and also a caution) is that any stray coupling between the resonator
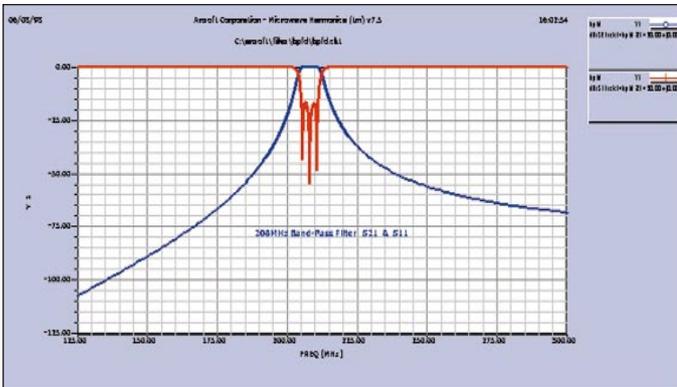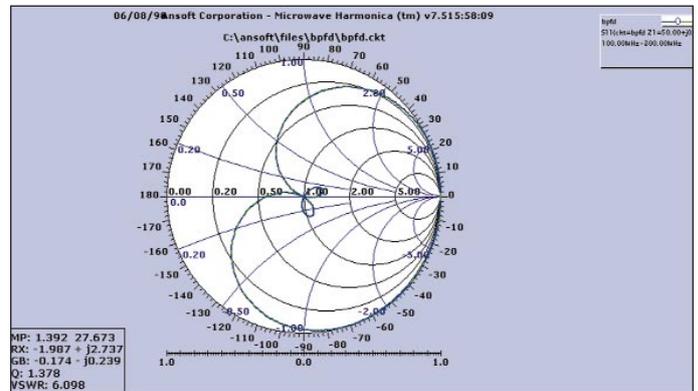
■ **Figure 4a. Pass-band and return loss for the 208 MHz filter example.**
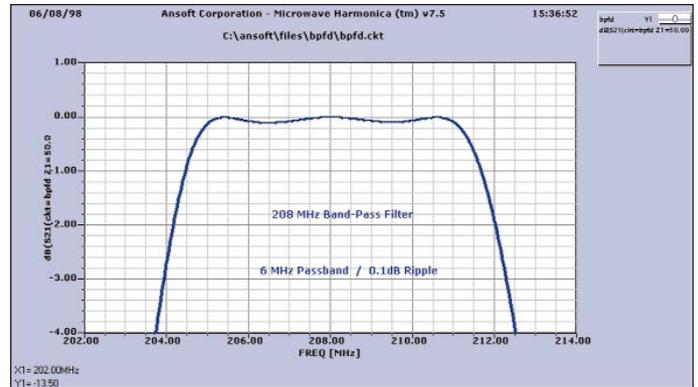
■ **Figure 4b. Ripple and pass-band detail for the 208 MHz filter example.**
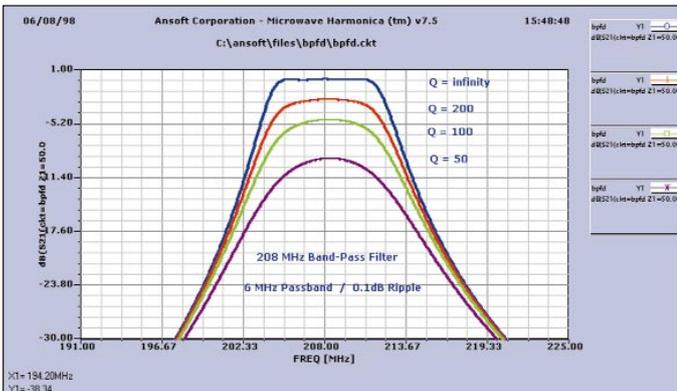
■ **Figure 4c. Effects of inductor Q for the 208 MHz filter example.**

■ **Figure 4d. Polar Smith chart of S_{11} (equal to S_{22}) for the 208 MHz filter example.**

sections (odd or even order) will cause wider bandwidths than designed for. This effect can be minimized with inductor shielding and careful component placement in the board layout.

An important concept to address is the out-of-band roll-off. Figure 4a confirms as mentioned earlier that roll-off is not as steep on the high end; 40-60 dB is commonly the limit. Adding to this problem are "fly-back" effects (Note 1). High Q components and a good layout should minimize the effects, but they eventually will come into play.

Where higher frequency roll-off is an issue, inductive coupling can help, providing a low-pass filter effect. Appendix II gives the derivation of two equations to do this. These equations (as well as the Appendix I and III derivations) were done with Laplacian transforms, and are fairly basic in nature.

An intuitive approach to the derivations may helpful, by seeing graphically what the equations are doing. The Smith chart in Figure 5 shows two paths going from a higher impedance (the result of Figures 1 and 3a) to a lower impedance (Figures 2 and 3b), typically 50 ohms. The higher path shows a route with a negative shunt capacitance $-C_p$, and series capacitance C_s. This is per the Appendix I Capacitive-Coupling derivation which converts to the standard configuration of Figures 2 and 3b.

The lower Smith chart path shows a positive shunt capacitance with a series inductor. This is the Appendix II Inductive-Coupling conversion, which can be done with either port (input or output) similar to the Capacitive-Coupling approach. Note, however, that this (lower Smith chart path) is a low-pass filter approach, whereas the upper path is a capacitive high-pass filter approach (see Note 2).

An example to increase higher frequency out-of-band roll-off with inductive coupling is shown in Figure 5 (next page). This will use the 208MHz filter example, and will convert the ports of figure 3-A to figure 3-C. Figure 6 shows the S21 pass-band. Note the roll-off improvement on the high end, trading off with the lower end. Also note the shape tends to be more symmetric or balanced.

Another helpful variation of the standard configuration is converting one or more of the shunt inductors to a series L/C resonator. As introduced above, this can especially knock out undesired frequency bands; and if the notch is set close in to the pass-band, the low end frequency roll-off is particularly steep. The derivation is given in Appendix III.

As it turns out, in the conversion process the resonant inductor value is larger than the single inductor it replaces. This works out well, because in especially narrow pass-band situations, small inductor (i.e. reactance) values are required. These are sometimes not practical
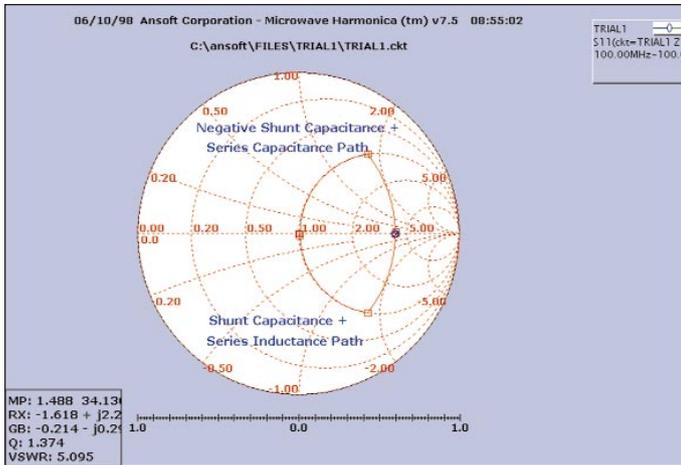
■ Figure 5. Capacitive and inductive coupling paths for the 208 MHz filter example.
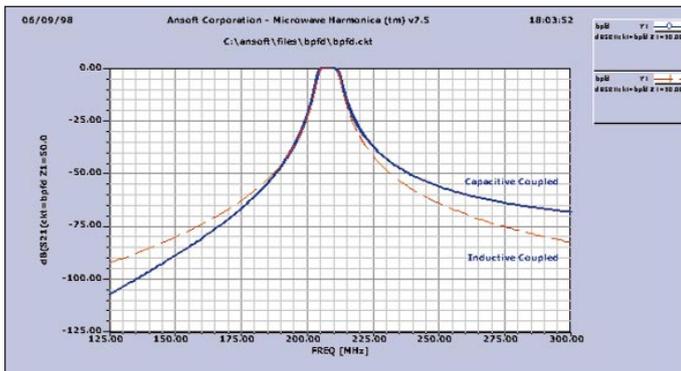
■ Figure 6. Analysis of Figure 3c for inductive coupled filter (inductive compared with capacitive coupled filter.)
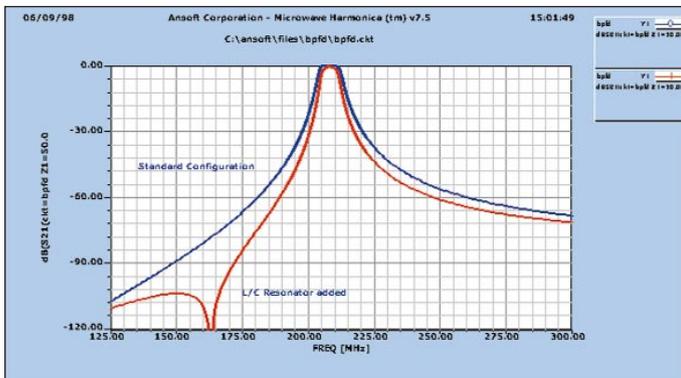
■ Figure 7. Pass-band plot of the L/C resonator in Figure 4c.

in value or convenient. But replacing the inductor with an L/C resonator allows (in fact requires) a larger value inductor. This can be set at the inductance desired; and the resonant notch frequency can also be independently fixed, by varying the series capacitor.

Applying this resonator process to the same 208 MHz example, assume there is an undesired image frequency 45 MHz below the center at 197.3MHz. The equations

and Figure 3d show the implementation of this final example. The Serenade Analysis results (Figure 7) show the deep notch as desired, using only one resonator. Also notice the pass-band has narrowed slightly, but the same basic shape has been maintained.

Although not done in this example, a considerably steeper roll-off close to the low pass-band side can be made, by bringing the notch closer towards the center frequency. In practical implementation however, the pass-band insertion loss will also increase, causing tradeoffs to be made. One final note is that the bandwidth of the notch is proportional to the L/C ratio, the impedance loading, and the number of resonators used.

Summary

Capacitive coupled filters often provide a practical solution to RF narrow-band filtering needs. The basic form has been presented here. This can be applied with lumped discrete components, or mixed with a distributed microstrip type form. Also modifications can be made in a straight forward fashion, which will add to the filtering capabilities. Particular variations have been presented in a practical form, for RF filter design enhancements.

Notes

(1) 'Fly-back' effect is defined here as inductors and capacitors that begin to resonate at the higher frequencies, and no longer electrically look inductive and capacitive. In fact, their reactances can completely switch. Electrically an inductor may look like a capacitor, and vice-versa. Therefore the signal may 'pop up' at some of the higher frequencies where the filtering action is lost. Ongoing studies are improving models for these effects.

(2) The lower shunt path could also be implemented with a negative shunt inductance and a series capacitor. Although not derived and detailed here, the approach is just as valid because there is already shunt inductance available to subtract the negative from.

References

1. A. Zverev, *Handbook of Filter Synthesis*, pp. 347-350.

2. Fred Taylor and Arthur Williams, *Electronic Filter Design Handbook*, pp. 5.19-5.24.

3. Nicola Volpicella, "Microwaves" April 1979, p. 125.

4. R. W. Rhea, *HF Filter Design and Computer Simulation,* p 1.

Author Information

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Appendix I — Capacitive-coupling conversion derivation: To go From a higher to a lower impedance
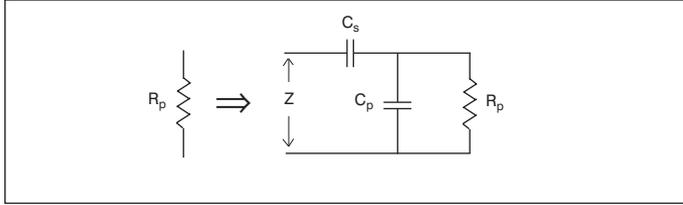
$Z(<R_p)$ is the new desired characteristic impendance
C_p is the negative capacitance.

$$Z = \frac{1}{SC_s} + \frac{1}{SC_p + \frac{1}{R_p}} = \frac{1}{SC_s} + \frac{R_p}{1 + R_p SC_p}$$

$$Z = \frac{1 + SR_p(C_p + C_s)}{SC_s + S^2 R_p C_s C_p}$$

$$SZC_s + S^2 z R_p C_s C_p - 1 - SR_p(C_p + C_s) = 0$$

$$j\omega Z C_s - \omega^2 Z R_p C_s C_p - 1 - j\omega R_p(C_p + C_s) = 0$$

$$\underbrace{1 + \omega^2 Z R_p C_s C_p}_{\text{set equal to zero}} + j\omega \underbrace{\left[Z C_s - R_p(C_p + C_s) \right]}_{\text{set equal to zero}} = 0$$

$Z\ sub\ C_p$ $\qquad ZC_s = R_p C_p + R_p C_s$

$$1 + \frac{\omega^2 Z R_p C_s{}^2 (Z - R_p)}{R_p} = 0 \qquad \boxed{C_p = \frac{C_s(Z - R_p)}{Rp}}$$
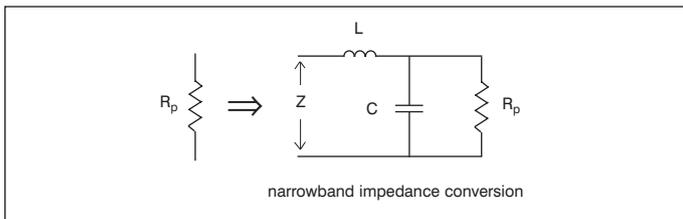
$$\qquad\qquad\qquad\qquad\qquad \text{2nd step Eq. (2)}$$

$$C_s{}^2 = \frac{-1}{\omega^2 Z(Z - R_p)}$$

$$\boxed{C_s = \left[\frac{1}{\omega^2 Z(R_p - Z)} \right]^{\frac{1}{2}}}$$

$$\text{1st step Eq. (1)}$$

Appendix II — Inductive-coupling conversion derivation: To go from a higher to a lower impedance

narrowband impedance conversion

$$\frac{\frac{R_p}{S_c}}{R_p + \frac{1}{S_c}} = \frac{R_p}{SR_p C + 1}$$

$$Z = S_c + \frac{R_p}{SR_p C + 1}$$

$$SZR_p C + Z = S^2 L R_p C = S_c + R_p$$

$$S(ZR_p C - L) + z - S^2 L R_p C - R_p = 0$$

$$\underbrace{jw(ZR_p C - L)}_{\equiv 0} + \underbrace{w^2 L R_p C + z - R_p}_{\equiv 0} = 0$$

$$\boxed{C = \frac{L}{ZR_p}} \qquad w^2 L R_p \frac{L}{R_p} + Z - R_p = 0$$

$$\text{Eq. 2}$$

$$L^2 = \frac{(R_p - Z)Z}{w^2}$$

$$\boxed{L = \left[\frac{(R_p - Z)Z}{w^2} \right]^{\frac{1}{2}}}$$

$$\text{Eq. 1}$$

Appendix III — Impedance Transformation Derivation: To Convert an Inductor to a Series L/C Resonance

$$SL_1 \equiv SL + \frac{1}{S_c} \qquad\qquad w_{o2}{}^2 = \frac{1}{L_c}$$

$$CS^2(L_1 - L) = 1 \qquad\qquad \boxed{C = \frac{1}{Lw_{o2}{}^2}}$$
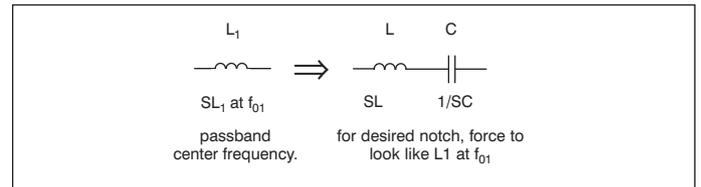
$$C = \frac{1}{S^2(L_1 - L)} \qquad\qquad\qquad \text{Eq. 2}$$

sub Equation 2 for C

$$\frac{1}{w_{o1}{}^2(L - L_1)} = \frac{1}{Lw_{o2}{}^2}$$

$$w_{o1}{}^2 L - w_{o1}{}^2 L_1 = Lw_{o1}{}^2$$

$$L(w_{o1}{}^2 - w_{o2}{}^2) = Lw_{o1}{}^2$$

$$\boxed{L = \frac{L_1 w_{o1}{}^2}{w_{o1}{}^2 - w_{o2}{}^2}}$$

$$\text{Eq. 1}$$

Appendix IV — Capacitive-coupled (CAPCPL) RF filter design program for Figure 2 configuration (written in program language C++)

```cpp
#include <stdio.h>
#include <conio.h>
#include <math.h>

float pi = 3.141592654;
float v= 299793000.;
float Am,beta,nu,r,p,fo,w,bw,z,zo,Cn,L,Qbp,Q,R,Cs,Cp,e,eff;
float W,A,B,h,t,lr,Zo,Ze,Wo,We,u,Rs,Rl,Cs1,Cs2,Cp1,Cp2;
float        a[40],b[40],c[40],C[40],g[40],q[40],k[40],K[40],
l[40],s[40];
int ct,n;

void main()
{
printf ("\n\n");
printf ("Enter Center Frequency (Hz) fo:");
scanf ("%f",&fo);
printf ("Enter Bandwidth (Hz) BW:");
scanf ("%f",&bw);
printf ("Enter Passband Ripple (dB):");
scanf ("%f",&Am);
printf ("Enter Number of Resonator Sections n:");
scanf ("%d",&n);
printf ("Enter Inductance L (nH):");
scanf ("%f",&L);
L = L*.000000001;
printf ("Enter Source Impedance (Ohms):");
scanf ("%f",&Rs);
printf ("Enter Load Impedance (Ohms):");
scanf ("%f",&Rl);
beta=log(1/(tanh(Am/17.37)));
nu=sinh(beta/(2*n));

for (ct=1;ct<n+1;ct++)
 {
 a[ct]=sin((2*ct-1)*pi/(2*n)); /* a[1] ... a[n] */
 b[ct]=nu*nu+pow(sin(ct*pi/n),2); /* b[1] ... b[n] */
 }
r=tanh(beta/4)*tanh(beta/4);
g[0]=1;        /* g[0] */
g[1]=2*a[1]/nu;    /* g[1] */

for (ct=2;ct<n+1;ct++) /* g[2] ... g[n] */
 { g[ct]=4*a[ct-1]*a[ct]/(b[ct-1]*g[ct-1]); }

if ((n&1)==0) /* even vs odd check for g[n+1] */
 { g[n+1]=1/r; } /* even check */
else
 { g[n+1]=1; }  /* odd check */

q[1]=g[0]*g[1]; /* q[1] */
q[n]=g[n]*g[n+1]; /* q[n] */
/* printf("q[1]= %f \n",q[1]); */
/* printf("q[%d]= %f \n",n,q[n]); */
for (ct=1;ct<n;ct++)
 {
 k[ct]=1/sqrt(g[ct]*g[ct+1]); /* k[1,2 ... k[n-1,n] */
/*  printf("k[%d,%d]= %f \n",ct,ct+1,k[ct]); */
 }
w = 2*pi*fo;
Qbp = fo/bw;
Q = Qbp*q[1];
printf ("Q= %f \n",Q);
Cn = 1/(w*w*L);
R = w*L*Q;
Cs1 = sqrt(1/(w*w*Rs*(R-Rs)));
Cp1 = Cs1*(Rs-R)/R;
Cs2 = sqrt(1/(w*w*Rl*(R-Rl)));
Cp2 = Cs2*(Rl-R)/R;
c[0] = 0;
c[n] = 0;

for (ct=1;ct<n;ct++)
 {
 K[ct]=k[ct]/Qbp; /* Coupling Coefficients */
 c[ct]=K[ct]*Cn;  /* Coupling Capacitances */
 }

for (ct=1;ct<n+1;ct++)  /* Resonator Capacitances */
 {
 C[ct]=Cn-c[ct-1]-c[ct];
 }

C[1] = C[1]+Cp1;
C[n] = C[n]+Cp2;
c[0] = Cs1;
c[n] = Cs2;

printf("L= %e \n",L);
for (ct=1;ct<n+1;ct++)
 { printf("C[%d]= %e \n",ct,C[ct]); }
for (ct=0;ct<n+1;ct++)
 { printf("C[%d,%d]= %e \n",ct,ct+1,c[ct]); }
 printf("\n press any key to exit...\n");
 getch();
}

90
```