Designer Tips:

Reactive Transformation of Resistances

Maximum power transfer is achieved when generator and load are matched. This article shows how to match using inductive and capacitive elements and how the resulting bandwidth relates to circuit Q.

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Maximum power transfer is achieved between two complex terminations when the real parts (resistances) have equal values. In the practical design of amplifiers and other circuits the generator and load resistances often have different values, and either an ideal transformer, lossless LC, or transmission- line network is needed to provide the necessary impedance transformation. The matching process may be grouped in two categories: one for resistive and another for complex terminations. This article covers the first category.

Impedance Matching of Resistive Terminations

Two uneven resistors always can be matched at a single frequency by a single LC section of one parallel and one series component, as shown in Figure 1, using the following procedure:

a. Add a parallel reactive element (inductor or capacitor) next to the larger resistor. The reactance, XP, is computed as

$$X_p = \frac{R_P}{Q} \qquad (1)$$

where RP is the resistance of larger resistor

RS is the resistance of the smaller resistor and

$$Q = Q_s = Q_p = \sqrt{\frac{R_p}{R_s} - 1} \qquad \text{(note: } R_P > R_S\text{)} \tag{2}$$

b. Add a series reactive element (inductor if the parallel element is a capacitor and capacitor if the parallel element is an inductor) to the lower resistor side, where the series reactance is defined as

$$Xs = RsQs$$

The LC section will provide simultaneous impedance transformation to both terminations; the larger termination will be down-transformed to the lower resistance and vice versa.

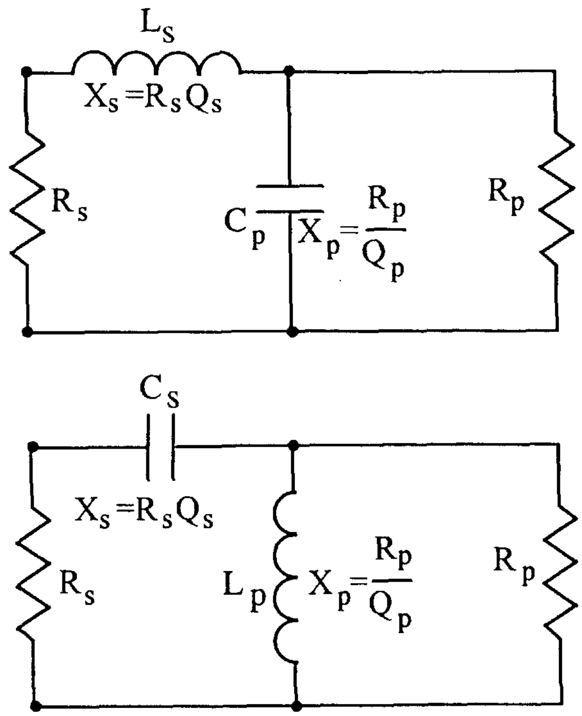

Figure 1. Lowpass and highpass circuit options to match two different resistive terminations. RS the smaller and RP is the larger resistor. The series and parallel sections have equal Q (Qs = Qp).

In the above reactance formulas, the Q = Qp = Qs is set by the ratio of the two terminations. If the resistor ratio is large, the Q will also be large and the resultant frequency response of the matching network will be quite narrow. For low-Q cases, (low Rp/Rs ratio) the network response will be wide. The relationship of the two resistors therefore defines the bandwidth, which may or may not be adequate. Therefore, the single-section matching have certain limitations. If the matching task is achieved by two or more sections, introducing intermediate impedance level, the termination ratios of the individual sections can be changed affecting the Qs. If the Qs are changed, the bandwidth is modified accordingly.

By choosing intermediate impedance levels between the two terminations (Rs < R-int < Rp), the termination ratios of both sections are lowered, therefore the bandwidth can be widened. If, on the other hand, the intermediate resistance level is selected to be outside the range of the two terminations (i.e. lower than the smaller resistor or higher than the higher resistor) the resultant frequency response will be narrower, providing a degree of filtering.

Illustrative Example

To illustrate this concept let's match a 5 ohm termination to 50 ohms, first using one LC section, and then repeat the same task with two two-section circuits for different goals: one for wider and one for narrower bandwidth. In all cases, the matching task will be centered at 400 MHz. Since the results will be later compared on a normalized Smith Chart, all computations are shown with normalized resistances and reactances, using Zo = 50 ohms. For simplicity, ideal components, (infinite Q, zero parasitics) are used.

Achieving the Match with a Single Lowpass LC Section

For convenience Figure 1 is redrawn, showing the normalized component values of both lowpass and highpass circuit sections for the 5-50 ohm matching example. The Q values of the series and parallel portions of the circuit are

$$Q_s = Q_p = \sqrt{\frac{r_p}{r_s} - 1} = \sqrt{\frac{1}{0.1} - 1} = 3 \tag{3}$$

The normalized reactances of the series and parallel elements of Figure 2 are

$$x_s = r_s Q_s = 0.1(3) = 0.3$$

$$x_p = \frac{r_p}{Q_p} = \frac{1}{3} = 0.333 \tag{4}$$

Selecting the lowpass option (series L and parallel C), the component values are

$$L_s = \frac{7.96 x_L}{f_{GHz}} = \frac{7.96 x_S}{f_{GHz}} = \frac{7.96(0.3)}{0.4} = 5.97\text{nH}$$

$$C_p = \frac{3.183}{f_{GHz} x_s} = \frac{3.183}{f_{GHz} x_p} = \frac{3.183}{(0.4)0.333} = 23.9\text{pF} \tag{5}$$

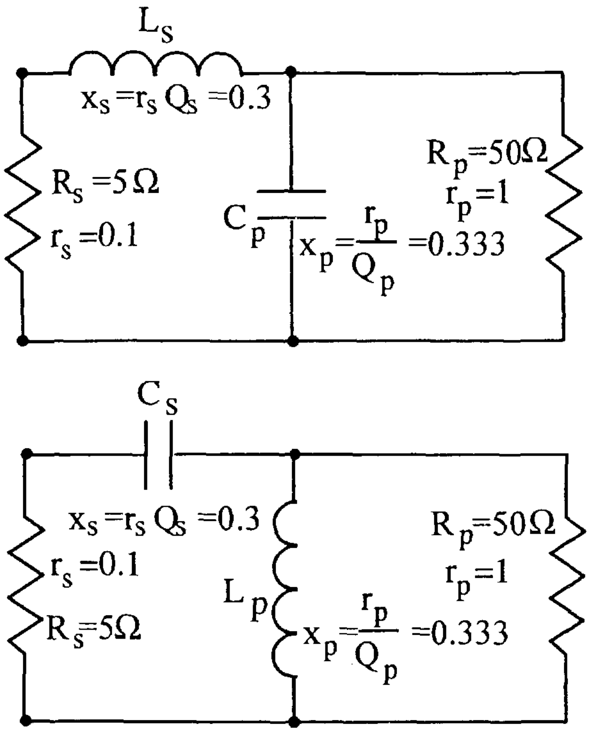

Figure 2. Normalized circuit diagrams, showing reactance values of the single lowpass and highpass matching sections, using rs = 0.1 and rp = 1.

The lowpass network provides an unsymmetrical frequency response at infinite frequency, the loss approaches infinity, while at DC the insertion loss is equal to the mismatch loss between the two resistors. The highpass section (series C and parallel L) would provide exactly the same impedance matching at 400MHz as the lowpass section. However, the broadband response is just the opposite—infinite attenuation at DC and a finite mismatch loss at infinite frequency.

Matching with Two Lowpass Sections for Wider Bandwidth

If the two source and load terminations are matched in two steps, the intermediate impedance level is chosen at

$$R_{int} = \sqrt{R_s R_p} \tag{6}$$

to lower the levels for equal component sensitivity. Of the two matching sections, the first to match 5 to 15.81 ohms. The second one matches 15.81 to 50 ohms. Both sections will have equal impedance matching ratios: 15.81/10 and 50/15.81. Each matching section may be of lowpass or highpass type, resulting in four possible combinations. For this illustration a lowpass-highpass mix is selected. Schematics of the circuit are shown in Figure 3.

The intermediate impedance level is

$$R_{int} = \sqrt{R_S R_L} = \sqrt{5(50)} = 15.81\Omega \quad \Rightarrow \quad r_{int} = \sqrt{r_S r_L} = \sqrt{0.1(1)} = 0.316 \tag{7}$$

The Qs of both sections are equal, since the termination ratios are the same.

$$Q_1 = \sqrt{\frac{r_{int}}{r_s} - 1} = \sqrt{\frac{0.316}{0.1} - 1} = 1.47$$

$$Q_2 = \sqrt{\frac{r_L}{r_{int}} - 1} = \sqrt{\frac{1}{0.316} - 1} = 1.47 \tag{8}$$

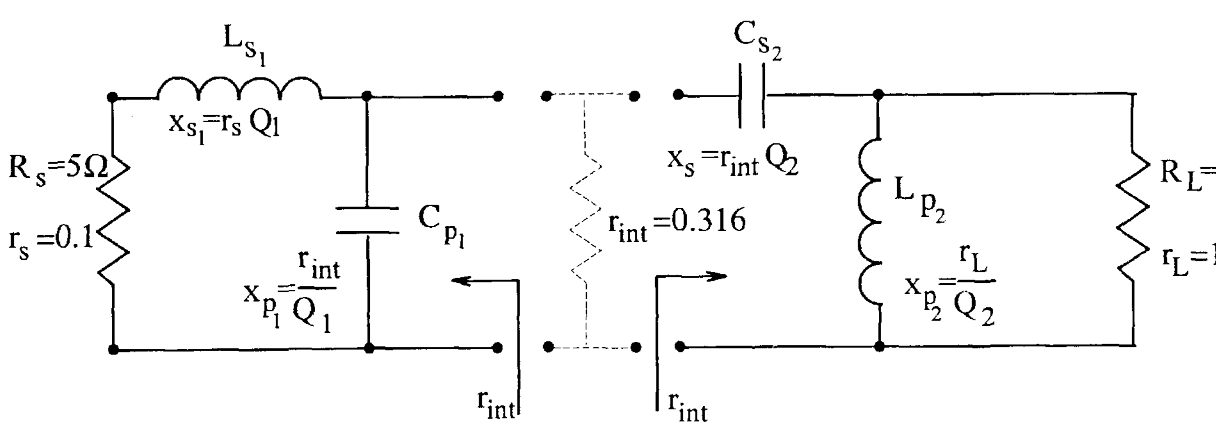

Figure 3. Two-section broadband matching network with one lowpass and one highpass section. The intermediate resistance level, r-int, is chosen to be the geometric mean of the two terminations.

The reactances of the two sections and the corresponding component values are computed at 400 MHz as

$$x_{s_1} = r_s Q_1 = 0.1(1.47) = 0.147 \qquad L_{s_1} = \frac{7.96 x_{s_1}}{f_{GHz}} = \frac{7.96(0.147)}{0.4} = 2.93\text{nH}$$

$$x_{p_1} = \frac{r_{int}}{Q_1} = \frac{0.316}{1.47} = 0.215 \qquad C_{p_1} = \frac{3.183}{f_{GHz} x_{p_1}} = \frac{3.183}{0.4(0.215)} = 37.36\text{pF}$$

$$x_{s_2} = r_{int} Q_2 = 0.316(1.47) = 0.465 \qquad C_{s_2} = \frac{3.183}{f_{GHz} x_{s_1}} = \frac{3.183}{0.4(4.65)} = 17.11\text{pF}$$

$$x_{p_2} = \frac{r_L}{Q_2} = \frac{1}{1.47} = 0.68 \qquad L_{p_2} = \frac{7.96 x_{p_2}}{f_{GHz}} = \frac{7.96(0.68)}{0.4} = 13.53\text{nH} \tag{9}$$

If the bandwidth is still not sufficient, additional section(s) may be used. However when real physical components are used, the component dissipations limit the maximum number of sections that can be used without excessive transformation loss. Also, with more sections the component Qs round the corners of the bandwidth, thereby also increasing loss.

Matching With Two Sections for Narrow Bandwidth

For narrow-band applications, the intermediate resistance level may be chosen either below the smaller resistor or above the larger resistor. Since impedance levels less than 5 ohms are difficult to reach, we select an intermediate resistance point above the 50 ohm value. The further this intermediate point is chosen from 50 ohms, the more selective the network response will be (we picked arbitrarily 130 ohms as the intermediate resistance level). As in the wideband example, four possible combinations exist: all lowpass, all highpass or mixing highpass and lowpass sections. One of these choices, containing one highpass and one lowpass is shown in the Figure 4. To be consistent with the previously used topologies, we start again with a lowpass circuit from the 5 ohm side, to be followed by a highpass section. The intermediate impedance level is higher than both terminations. Of the two sections, the Q of the left side (lowpass) section will be higher since its termination ratio is higher than the highpass section's. The Q of the first section ($Q1 = 5$) is the main reason for the reduction of the bandwidth of the circuit. Further reduction can be reached with higher Q1, by setting the intermediate impedance level higher. The intermediate impedance level and the two resulting circuit Qs are:

$$R_{int} = 130\Omega \quad \Rightarrow \quad r_{int} = 2.6$$

$$Q_1 = \sqrt{\frac{r_{int}}{r_s} - 1} = \sqrt{\frac{2.6}{0.1} - 1} = 5 \qquad Q_2 = \sqrt{\frac{r_{int}}{r_L} - 1} = \sqrt{\frac{2.6}{1} - 1} = 1.26 \tag{10}$$

The reactances of the components and the component values of the circuit in Figure 4 are now computed as

$$x_{s_1} = r_s Q_1 = 0.1(5) = 0.5 \qquad L_{s_1} = \frac{7.96 x_{s_1}}{f_{GHz}} = \frac{7.96(0.5)}{0.4} = 9.95\text{nH}$$

$$x_{p_1} = \frac{r_{int}}{Q_1} = \frac{2.6}{5} = 0.52 \qquad C_{p_1} = \frac{3.183}{f_{GHz} x_{p_1}} = \frac{3.183}{0.4(0.52)} = 15.3\text{pF}$$

$$x_{p_2} = \frac{r_{int}}{Q_2} = \frac{2.6}{1.26} = 2.06 \qquad L_{p_2} = \frac{7.96 x_{p_2}}{f_{GHz}} = \frac{7.96(2.06)}{0.4} = 41.06\text{nH}$$

$$x_{s_2} = r_L Q_2 = 1(1.26) = 1.26 \qquad C_{s_2} = \frac{3.183}{f_{GHz} x_{s_1}} = \frac{3.183}{0.4(1.26)} = 6.32\text{pF} \tag{11}$$

Since both matching sections have a parallel component at the intermediate point, those two elements can be combined together forming a Tee network with the two series arms. Since the two parallel elements are different types—one capacitor and one inductor—the replacement is their combined equivalence at the center frequency. The

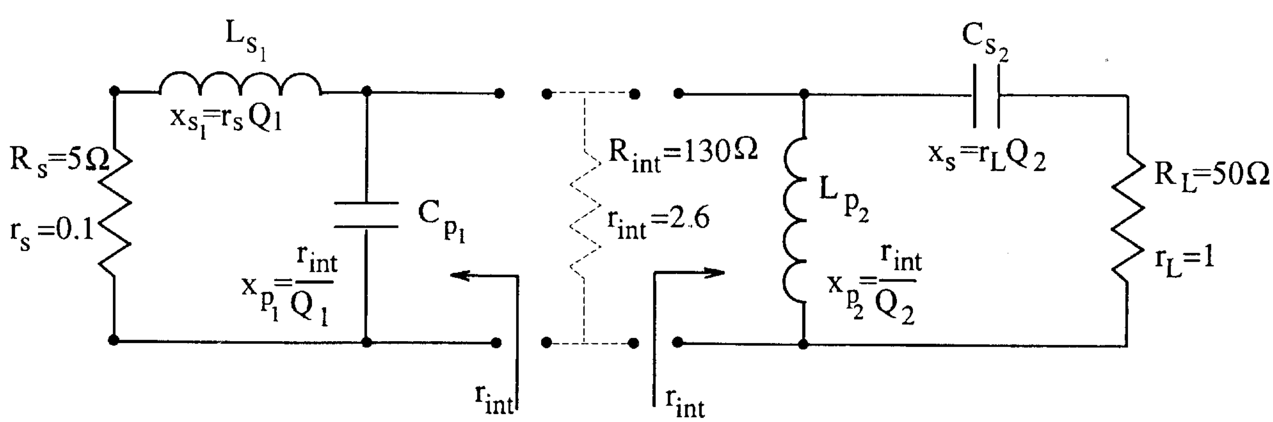

Figure 4. Circuit schematics of the two-section narrowband matching network. For maximum symmetry, equal numbers of highpass and lowpass sections are used.

normalized reactance of this single component is found as

$$x_{eq} = \frac{jx_L(-jx_C)}{jx_L - jx_C} = \frac{j2.06(-j0.52)}{j2.06 - j0.52} = -j0.696 \quad (12)$$

The equivalent capacitor at 400MHz is given by:

$$C_{peq} = \frac{3.183}{(0.4)0.696} = 11.43\text{pF} \quad (13)$$

Therefore, the 41.06nH parallel inductor and the 15.3pF parallel capacitor may be replaced with an 11.43pF capacitor, with only minor change of insertion loss between 300MHz and 500MHz. If the intermediate resistance had been chosen below 5 ohms, the two networks would have had different topologies with such configurations that, after combining, form a PI network.

Smith Chart Analysis of the Three Networks

Next, let's compare the three transformations on the Smith Chart of Figure 5a, that includes constant-Q contours of 0.5, 1.5, 3 and 5. The single section circuit shows a maximum Q value of 3, the highest Q of the wideband circuit is 1.47, while the narrowband circuit Q reaches a maximum value of 5. At the design center frequency of 400 MHz all three circuits would provide a perfect match between 5 ohms and 50 ohms. However, when the frequency is changed to 500MHz, the responses are quite different, as displayed on the Smith Chart of Figure 5b. Both plots were generated using the WINDOWS-based Smith Chart program called EEZMatch[TM].

Although the Smith Chart is basically a reflection coefficient analysis tool, the insertion loss of a lossless two-port can be computed from the reflective mismatch loss. In the case of lossless two-ports, the signal applied to the input is either reflected or transmitted. The sum of the reflected power and transmitted power is equal to the total power applied.

$$|s_{11}|^2 + |s_{21}|^2 = 1 \quad (14)$$

Therefore, the power gain is:

$$|s_{21}|^2 = 1 - |s_{11}|^2 \quad (15)$$

Or, since the insertion loss is the reciprocal

$$\text{Insertion loss} = \frac{1}{|s_{21}|^2} = \frac{1}{1 - |s_{11}|^2} \quad (16)$$

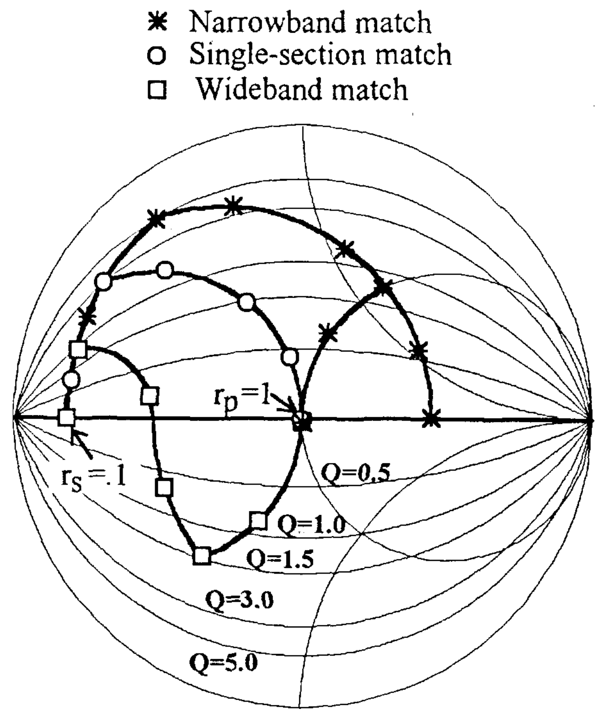

Figure 5a. Immitance loci of the three matching networks at 400Mhz, showing perfect match for each case. Note the parallel resonance phenomena of the parallel LC portion of the narrowband circuit.

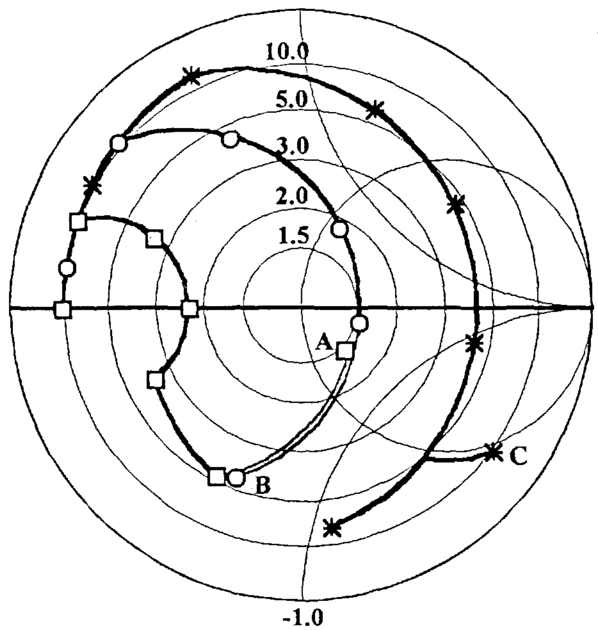

Figure 5b. At 500MHz the wideband circuit (A) still shows good match (/Gamma/ = 0.23), but the other two circuits have significantly larger reflection coefficients: 0.54 for the single-section (B), and 0.80 for the narrowband circuits (C).

The reflection coefficients at 500MHz may be converted to VSWR and mismatch loss, as tabulated in Table 1. Conversion formulas are:

$$\text{VSWR} = \frac{1 + |\Omega|}{1 - |\Omega|} \quad (17)$$

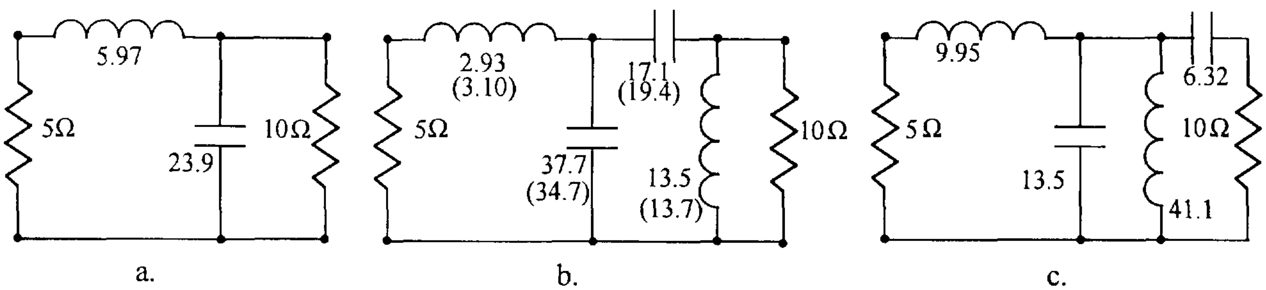

Figure 6. Component values of the three matching circuits, using ideal elements. a. Single-section, b. Wideband, c. Narrowband topologies. Inductor and capacitor values are shown in nanohenries and picofarads. Components values in parentheses show optimized equal-ripple wideband results shown in Figure 7.

and

$$ML(dB) = -10\log(1-|S_{22}|^2) \tag{18}$$

Parameters	Single-section circuit	Wideband circuit	Narrowband circuit
/s22/	0.64	0.23	0.81
VSWR	4.55	1.60	9.70
Insertion loss	2.25dB	0.23dB	4.73dB

Table 1. Comparison of output parameters and computed insertion loss at 500MHz.

For comparison, all three circuits were analyzed using SuperCompact[TM] over the 300-500MHz bandwidth. Schematics and the corresponding insertion loss plot are displayed in Figures 6 and 7, showing the noticeable differences in selectivity.

The parallel LC resonant section of the narrowband circuit (Figure 6c) may be replaced with the equivalent dominant element (single capacitor) at 400MHz, resulting a slight change in selectivity, a deletion well worth the small change in performance.

From the foregoing it is clear that the 3dB bandwidth of single series or parallel bandpass resonator networks can be computed exactly. Predicting the response of bandpass matching networks of arbitrary configuration by hand calculation is a far more difficult task, better relegated to CAE techniques. What is important is the technical insight that any two resistors can be matched at one frequency using the simple computations outlined above. The circuit's frequency response then can be adjusted widely by changing the Q(s) of the matching section(s).

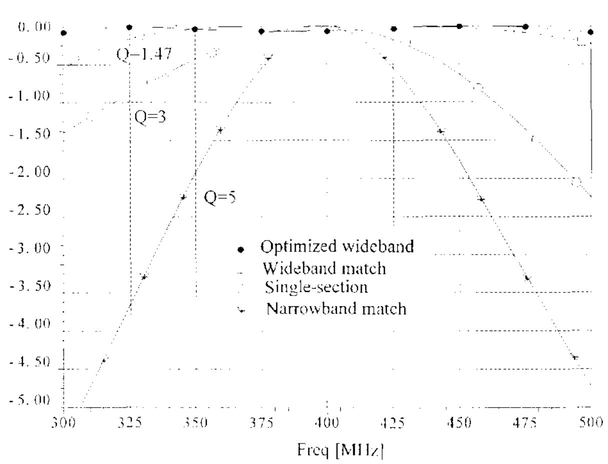

Figure 7. Insertion loss plots in dB for the three matching circuits of Figure 6, showing the dependency on section Q. Solid markers indicate optimized results.

References

1. C. Bowick, RF Circuit Design, Prentice Hall, 1982.
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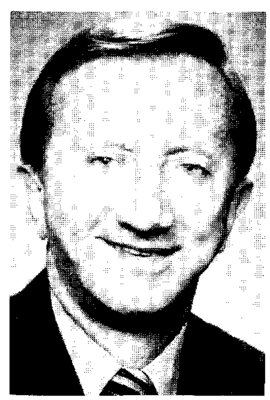

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