PRACTICAL MICROWAVE FILTER DESIGN

Modern design of microwave filters has evolved to a specialization, yet it can be handled in a step-by-step fashion, as this author shows.

by Michael Ferrand
Microlab/FXR, Livingston, NJ

Introduction

The design of filters is a specialized area of microwave engineering that is a daunting first experience. However, when carried out in a step by step basis, it can be handled in a straight-forward manner. This paper shows the steps to designing a waveguide bandpass filter using impedance inverters, along with practical results.

The Lowpass Prototype

Starting with the calculation of the number of sections required we will step thru a design for a waveguide filter. Several parameters are used to characterize a filter's performance. The starting requirement most often specified is frequency response. When given a frequency-response specification, the designer must select a design that meets these requirements. This is accomplished by transforming the required response to a normalized lowpass specification having a cutoff frequency of 1 radian/second.

This normalized response curve may be compared with curves for lowpass prototype filters (which also have 1 rad/sec cutoff. From these curves a prototype filter of the proper order (number of sections) may be designated. Curves for the normalized frequency verses amplitude response have been tabulated for many families of filters, each having auxiliary characteristics which influence the design choice, as will be shown. Reference 1 gives perhaps the clearest curves. The tabulated response families and their characteristics are:

Butterworth:also known as maximally flat

Chebyshev: equal-ripple amplitude response in passband

Bessel: linear phase

Gaussian: linear or equal-ripple delay response

Legendre: least-squares amplitude response

Elliptic: equal ripple amplitude response in passband and stopband

The most commonly used response families are the Butterworth and Chebyshev. The equations to calculate the response for these two types are given in [1] and Figure 1.

Impedance and Frequency Scaling

The basis for normalization of filters is the fact that a given filter's response can be scaled to a different frequency range by dividing the reactive elements by a scaling factor. For example, if you have a filter design with a 1 Hz cutoff frequency which uses inductances in henrys and capacitors in farads, you can scale it to 1 GHz by using the same numeric values for the elements but changing their dimensions to nanohenries and nanofarads respectively. For any other case, the frequency scaling factor is just the ratio of the reference frequency of the desired filter to corresponding reference frequency of the particular prototype filter.

Except for Elliptic and Chebyshev filters the 3 dB points on the edge of the passband are selected as the cutoff. In the case of our lowpass prototype with the radian cutoff frequency normalized to one we specify a stopband frequency and corresponding attenuation value at that frequency. For example, one could select a 0.01 dB passband ripple Chebyshev with a stopband frequency that is 2.4 times the cutoff, at which the attenuation is to be 60 dB.

The required order may be calculated or read from the curve in [2]. We find that a 6 section filter will satisfy the requirement. In practice, one usually selects an order that is higher than required to meet the specified attenuation and allow for dissipative losses and manufacturing tolerances that would otherwise cause the practical filter execution to fall short of the theoretically achievable performance.

The calculation of the required number of sections for a bandpass is quite similar to that of the lowpass filter. We may use the bandwidth of the bandpass filter normalized to 1 rad/sec. and relate it to a normalized stopband bandwidth. For example if we set our normalized bandwidth to 1 and have 60 dB attenuation at 2.4 rad/sec, then our ratio is 2.4. We can use this value, along with the desired

atenuation level (60 dB) to calculate the number of sections needed.

Design by Normalized Element Values

We can relate a normalized frequency response to our filter and calculate the required order of the filter. We may also design our prototype filter through the use of normalized element values. By this method we set the lowpass cutoff frequency to 1 rad/sec and the source and load resistances to 1 ohm. After the calculation of the lowpass prototype filter's element values, we frequency and impedance scale the lowpass filter to the desired final frequency and impedance.

Fortunately there are published tables of these element values[2,3]. For convenience, the equations for Chebyshev and Butterworth filters are included in Figure 2. Having calculated the required number of sections, it merely is required to consult the appropriate table to read the element values.

Waveguide Filter Design

The procedure is easier to follow with an actual example. For this example, choose a filter to be realized in WR-75 waveguide having:
Passband: 15.675 to 16.425 GHz.
Stopband/Attenuation: 45 dB. minimum at 14.950 GHz.
45 dB. minimum at 17.000 GHz.
Passband Insertion Loss: 0.5 dB. max.
Passband VSWR: 1.3:1 max.

The most common filtering function used is the Chebyshev response. This gives the most rectangular passband response. We will choose a very low ripple design, say 0.1 dB, to assure that there are no large amplitude variations in the passband. Using these specifications we will design our device from the procedures described above via the following steps.

1. Calculate the filter bandwidth (BW), 16.425 - 15.675 = 0.750 GHz.

2. Calculate the filter center frequency (Fo), 15.675 + (0.75/2) = 16.050 GHz.

3. Calculate the stopband width by, 2(Fo-14.95) = 2.20 and 2(17.00 - Fo) = 1.900 GHz.

For Butterworth response ...

$$n = \frac{LOG\left[\dfrac{10^{adB/10}-1}{\epsilon}\right]}{2\,LOG\left[w\right]+1}$$

where adB is required attenuation

w is the stopband / passband ratio

$$\epsilon = \sqrt{10^{3/10}}$$

For Chebyshev response ...

$$n = ACOSH\left[\frac{\sqrt{10^{adB/10}-1}}{\sqrt{10^{R/10}-1}}\right] \bigg/ ACOSH\left[w\right]+1$$

where adB is required attenuation

w is the stopband / passband ratio

r is the dB. ripple

Figure 1. The formulas used to calculate Butterworth and Chebyshev insertion loss as a function of the normalized frequency.

4. Calculate a normalized stopband/passband ratio from the previous values. For the upper stopband, 1.90/0.750 = 2.533, we will call this the normalized 45 dB bandwidth. Then for the lower stopband, 2.20/0.750 = 2.933. We can see that the upper stopband represents the tighter requirement.

5. Using Figure 1 or the charts in [3] calculate the necessary order, or number of poles. We calculate 5 poles necessary for 2.533 ratio and 4 poles for the 2.933 ratio, choosing the higher integer number of poles to assure that the attenuation requirements are met.

We have now found that the waveguide filter must have a minimum of 5 poles to meet our stopband/attenuation requirements. With this information we continue the design using the normalized lowpass element values. Again using[3] (or Figure 2) we apply the tables for a 0.1 dB Chebyshev design. Reference to the table yields:

RL = 1.0000
C1 = 1.1468
L2 = 1.3712
C3 = 1.9750
L4 = 1.3712
C5 = 1.1468
RS = 1.0000

The fact that these values are symmetrical suggests that the final design will be, too. Note that these normalized element values are conventionally designated as G1, G2, G3 ... in other references. We will use this convention also.

Inverters

Filter designs using inverters are the most common in the microwave frequency range. Direct-coupled filters may be found in very low frequency through the upper microwave frequencies, often in such different forms as to appear totally unrelated. The formal basis for the use of inverter networks is attributed to Dishal (1949) and Cohn (1957). In it's simplest form an inverter may be represented as a quarter wavelength of transmission line. Inverters in combination with other reactive elements will help us to realize a bandpass filter at microwave frequencies. In it's simplest form an inverter may be considered a lossless quarter wavelength of transmission line. The inverter will help us directly convert our normalized lowpass filter to the microwave bandpass filter we wish to design. The use of inverters is simple and their descriptions may be found in [1] thru [3].

The waveguide filter serves as an excellent inverter design example. The filter consists of shunt inductive posts separated by lengths of transmission line (in this case lengths of waveguide). The actual design is carried following the procedure given in [3], and Figure 3.

1. Calculate the guide wavelengths per step 1 of Figure 3.
w-Lambda   = .072228
Lambda-g0   = .913415
Lambda-g1   = .947800
Lambda-g2   = .881826
2. Now calculate the inverter values from step 2 of Figure 3.
K0,1/Zo = .314535
K1,2/Zo = .090476
K2,3/Zo = .068943
K3,4/Zo = .068943
K4,5/Zo = .090476
K5,6/Zo = .314535
3. Now the shunt reactances Xi,i+1/Zo from step 3 of Figure 3.
X0,1/Zo = .348731
X1,2/Zo = .091223
X2,3/Zo = .069272
X3,4/Zo = .069272

for Butterworth response

$$G_0 = 1$$

$$G_k = 2 \, SIN \left[\frac{(2^k - 1)\pi}{2^n} \right] \qquad k = 1 \; to \; n$$

$$G_{n+1} = 1$$

for Chebyshev response

$$B = LOG \left[COTH \frac{r}{17.37} \right] \qquad where \; r = dB. \; Ripple$$

$$\gamma = SINH \frac{B}{2n}$$

$$a_k = SIN \left[\frac{(2k-1)\pi}{2n} \right] \qquad k = 1 \; to \; n$$

$$b_k = \gamma^2 + SIN^2 \left(\frac{k\pi}{n} \right) \qquad k = 1 \; to \; n$$

$$G_1 = \frac{2a_1}{\gamma}$$

$$G_k = \frac{4 a_{k-1} a_k}{b_{k-1} G_{k-1}} \qquad k = 2 \; to \; n$$

$$G_{n+1} = 1 \qquad for \; n \; odd$$

$$COTH^2 \left(\frac{B}{4} \right) \qquad for \; n \; even$$

Figure 2. The equations used to calculate the prototype element values, g, for Butterworth and Chebyshev filter responses.

X4,5/Zo = .091223
X5,6/Zo = .348731
4. Now calculate the post spacings, Theta from step 3.
Theta(1) = 2.74700 radians
Theta(2) = 2.98278
Theta(3) = 3.00414
Theta(4) = 2.98278
Theta(5) = 2.74700
We need to calculate the ratio of the post diameter to the guide width, d/a, to produce the required normalized reactance X/Zo for each post. To get the post diameters use a published chart such as [3] Fig.8.06-3(a). For very thick posts there is a correction in the electrical reference plane for the post. Where this correction is needed, refer to [3] Figure 8.03-3(a) and equation 5b. In this case we will ignore the correction. A computer program which is available to readers automates this correction, more about the program later.

$$\lambda_{g_0} = \frac{1}{\sqrt{(0.08472\, f_0)^2 - \left(\frac{1}{2a}\right)^2}}$$

$$\lambda_0 = \frac{11.803}{f_0}$$

$$\lambda_{g_1} = \frac{1}{\sqrt{(0.08472\, f_1)^2 - \left(\frac{1}{2a}\right)^2}}$$

$$w\lambda = \frac{\lambda_{g_2} - \lambda_{g_1}}{\lambda_{g_0}}$$

$$\lambda_{g_2} = \frac{1}{\sqrt{(0.08472\, f_2)^2 - \left(\frac{1}{2a}\right)^2}}$$

STEP 2. Calculate impedance inverters

$$\frac{K_{0,1}}{Z_0} = \sqrt{\frac{\pi}{2} \cdot \frac{w\lambda}{G_0\, G_1}}$$

$$\frac{K_{n,n+1}}{Z_0} = \sqrt{\frac{\pi}{2} \cdot \frac{w\lambda}{G_n\, G_{n+1}}}$$

$$\frac{K_{j,j+1}}{Z_0} = \frac{\pi w\lambda}{2} \cdot \frac{1}{\sqrt{G_j\, G_{j+1}}}$$

STEP 3. Calculate shunt reactances

$$\frac{X_{j,j+1}}{Z_0} = \frac{\dfrac{K_{j,j+1}}{Z_0}}{1 - \left(\dfrac{K_{j,j+1}}{Z_0}\right)^2}$$

$$\theta_j = \pi - .5 \left[\tan^{-1}\left(2 \cdot \frac{X_{j-1,j}}{Z_0}\right) + \tan^{-1}\left(2 \cdot \frac{X_{j,j+1}}{Z_0}\right)\right]$$

Figure 3. Guide wavelength, impedance inverter and shunt reactance calculations.

Using such a chart or the equations on which it is based, the normalized post diameters are calculated and the results multiplied by the

the design involves some approximations . . . the final response usually can be compensated by tuning adjustments

waveguide width, for this design a = 0.750 inches, giving for the respective post diameters the following values:
Post(1) = .07145 inches
Post(2) = .14636
Post(3) = .16032
Post(4) = .16032
Post(5) = .14636
Post(6) = .07145
and the post spacings, converted from radians to inches,
Spacing(1,2) = .39992 inches
Spacing(2,3) = .43425
Spacing(3,4) = .43425
Spacing(4,5) = .39992
Given the post diameters and their spacings in the specified waveguide, WR 75, the filter design is completed from a theoretical standpoint.

Results

The measured results of the device are plotted in Figure 4 and a photograph of the model is shown in Figure 5. Although the design involves some approximations, step 2 concerning the inverter values, the final response usually can be com-

The design equations . . . are easily implemented in a computer program

pensated by tuning adjustments, as was the case with this model. Tuning screws, size 2-56, were located on the waveguide broadwall midway between the posts. The data shown were obtained after judicious tweaking with the screws, to make the resonance of each section of the filter
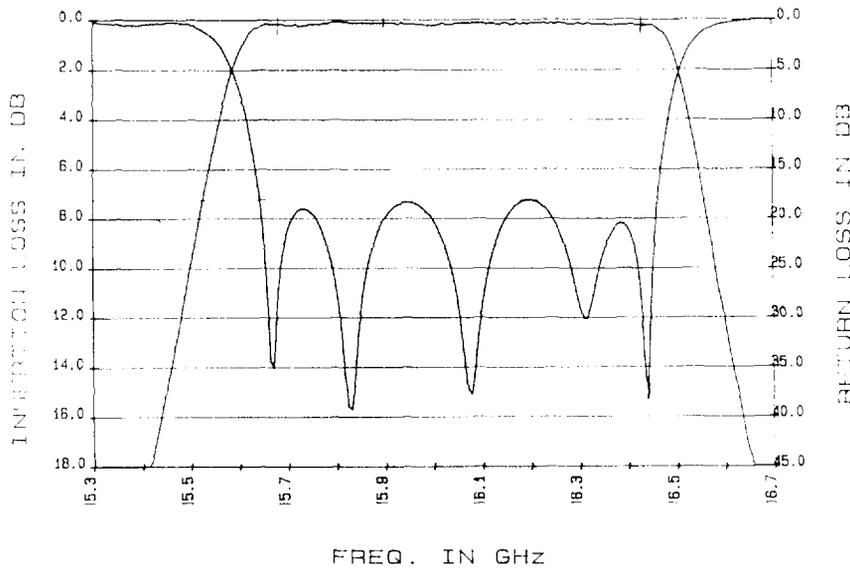
FREQ. IN GHz

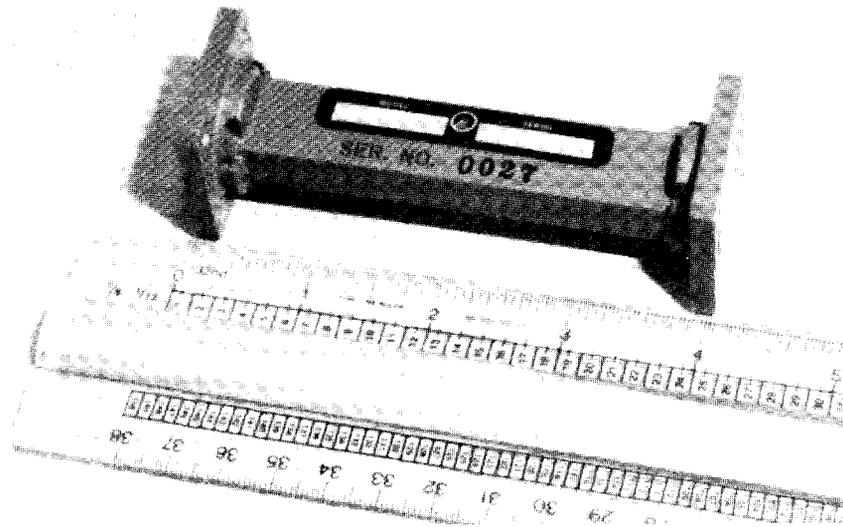**Figure 4. Insertion loss and return loss measurements for the waveguide filter model.**

equal to the center frequency of the design. Begin at either end and work through the filter. There is very little interaction with the adjustments.

The design equations given in Figure 2 are easily implemented in a computer program which can eliminate the tedious calculations. The program also builds in the correction for thick posts mentioned earlier. A program based on the design equations used herein and approximations for the post diameters and spacings given in [4] is available from the author. For a handling charge of $25.00 the FORTRAN source code and an executable file will be sent on a 360K, 5 ¼ inch, MS-DOS floppy disk suitable for use on an IBM compatible XT or AT personal computer.

References

1. S.B. Cohn Parallel-Coupled Transmission Line Resonator Filters IRE Trans. MTT Apr. 1958
2. G.L. Matthaei, L. Young, EMT Jones. Microwave Filters, Impedance Matching Networks and Coupling Structures. McGraw-Hill, 1964.
3. A. Zverev — Handbook of Filter Synthesis. J. Wiley and Sons, 1967.
4. S/FILSYN — Filter synthesis program by Dr. George Szentermai, DGS Associates.

Figure 5. Photograph of the 5 section WR-75 waveguide filter.

Michael K. Ferrand *is a senior microwave engineer with Microlab/FXR in Livingston, New Jersey. His responsibilities include the design of microwave filters and other microwave components, and he has* specialized in the computer-aided design of these components. He is the author of several papers on filter designs based on the use of the computer.