

A Lumped Element Rat Race Coupler

At most frequencies distributed element hybrids, such as the rat race coupler, consume too much valuable circuit real estate, particularly in MMICs. Lumped element designs greatly reduce size without large performance costs.

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The conventional 180 degree, 3 dB transmission line hybrid, often called a "rat-race coupler" or a "ring hybrid" is depicted in Figure 1 together with its electrical characteristics normalized to a 1 Hertz center frequency. Due to symmetry, the return loss behavior of ports 3 and 4 are not shown. This abbreviated presentational procedure will be used throughout the paper. At frequencies below 18 GHz, this hybrid occupies much more area than an equivalent lumped element design. For example, a 70.7 ohm, quarter wavelength transmission line on a 100 micrometers thick GaAs substrate at 8 GHz is 3328 micrometers in length. When realized in ring form, it occupies approximately 32 square millimeters in this medium. An electrically equivalent design (at the center frequency) in lumped element form occupies only about 1 square millimeter, an area savings of over 96%, i.e. a reduction in area by a factor of 30!

Nor is the lumped approach only useful to reduce the occupied area of couplers in MMIC circuits. The same methods can be employed to shrink coupler sizes in stripline or microstrip, particularly below 1 GHz, for which frequencies the wavelength, even in dielectric, can exceed 1 foot.

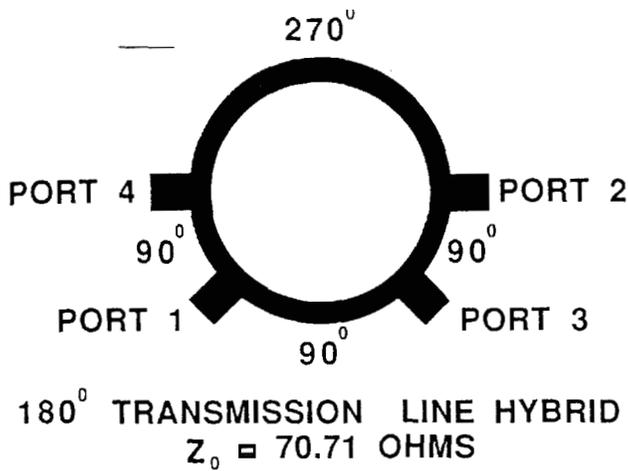


Figure 1a. The distributed element "rat race" coupler.

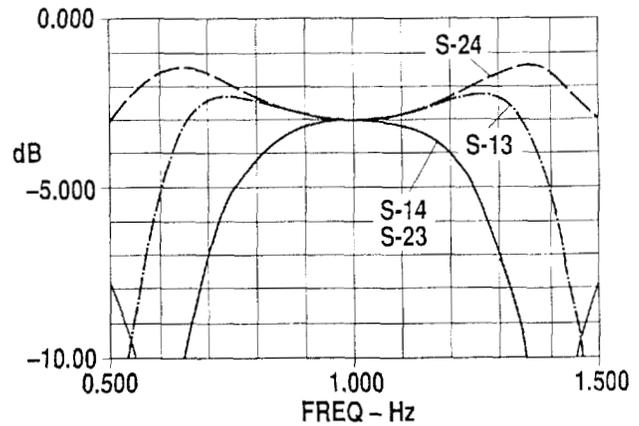


Figure 1b. Coupling versus frequency of the rat race.

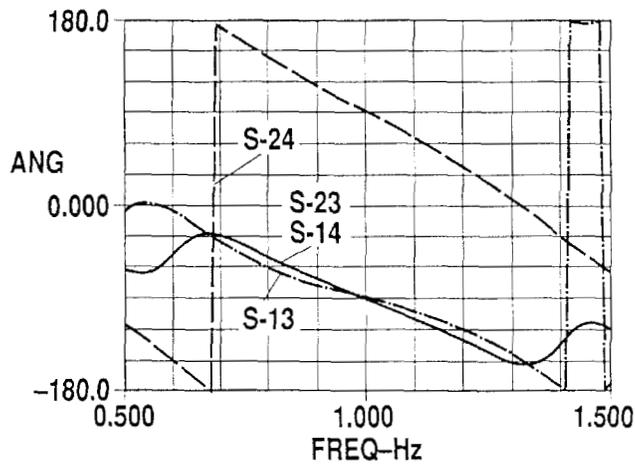


Figure 1c. Phase versus frequency of the outputs.

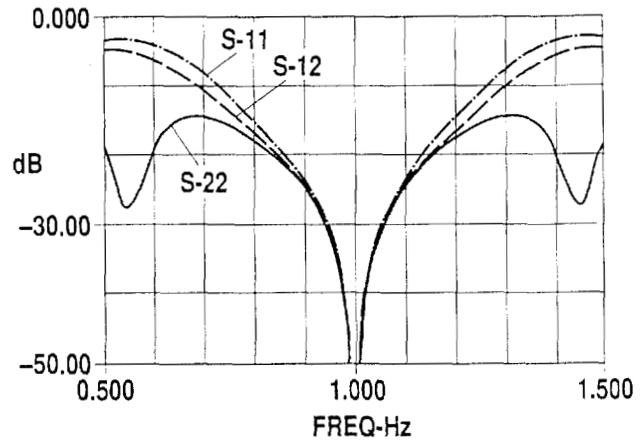


Figure 1d. Isolation and return loss versus frequency.

The basis of the design is to derive equivalent "pi" and "tee" networks for the transmission line segments of the "rat-race". From a theoretical basis, this is accomplished by writing the ABCD matrix for the corresponding parts of the distributed and lumped element hybrids and equating the corresponding respective terms [1, pg. 229].

The same methods can be employed to shrink coupler sizes in stripline or microstrip

This procedure then gives relationships that must be satisfied if the lumped and distributed circuits are to be equivalent at the design center frequency. The equivalence, of course, applies perfectly at only

the center frequency of the design; however, as will be seen, the lumped equivalent is good enough to provide the modest bandwidth often needed for most applications.

The basis of the design is to derive equivalent "pi" and "tee" networks for the transmission line segments

The quarter wavelength, or 90 degree, transmission line segments were modeled as low pass "pi" networks. The result of the analysis is that the impedance of the elements in the low pass network is numerically equal to the impedance of the quarter wave line section being replaced. Thus, for example, a lumped pi equivalent of a 90 degree, 70.7 ohm

characteristic impedance line section has inductors and capacitors whose reactances are also 70.7 ohms in magnitude at the frequency for which the equivalence applies.

A "pi" equivalent is preferred to a "tee" in an integrated circuit application because it requires fewer inductors for its realization. For a planar layout, the spiral inductors used exhibit relatively high resistive losses, consequently their use should be minimized in lumped element designs.

The equivalence . . . applies perfectly at only the center frequency of the design;

The three quarter wavelength, 270 degree, transmission line section of the distributed hybrid is replaced in the lumped design with a high-pass "tee" network since the high pass "tee" requires fewer inductors than would a high-pass "pi" network. The matrix operations comprising the analysis of the circuits are shown in the Appendix. Similarly, for the high pass circuit, the reactances of the elements are also equal numerically to the characteristic impedance of the 270 degree line length being replaced, at the frequency at which the equivalence applies.

The final circuit configuration is shown schematically in Figure 2, along with its calculated performance as functions of normalized frequency.

Table I lists the component values required (reactances of 70.7 ohms) for lumped element hybrids at frequencies as widely disparate as 80 MHz

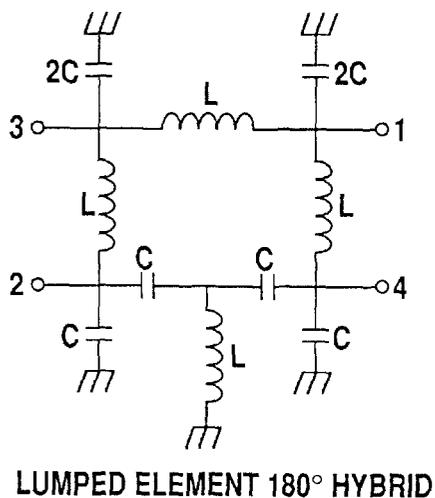


Figure 2a. The lumped element rat race equivalent circuit.

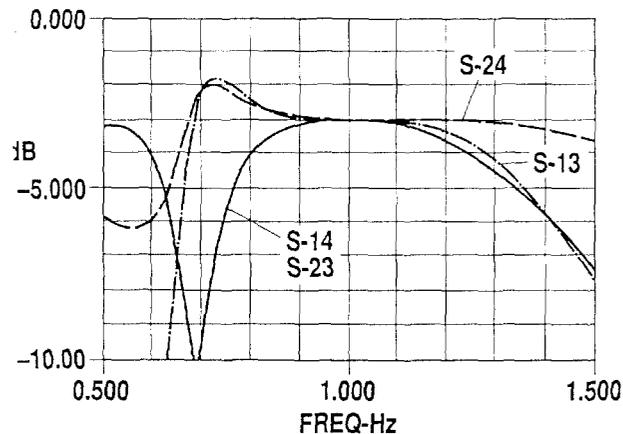


Figure 2b. Calculated coupling versus frequency for the lumped equivalent.

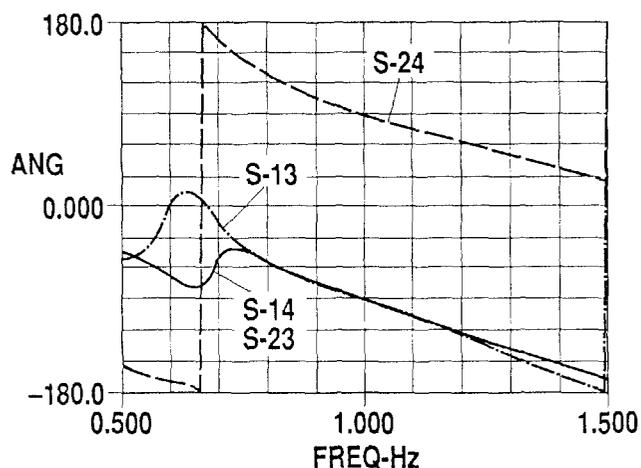


Figure 2c. Calculated phase response for the lumped circuit.

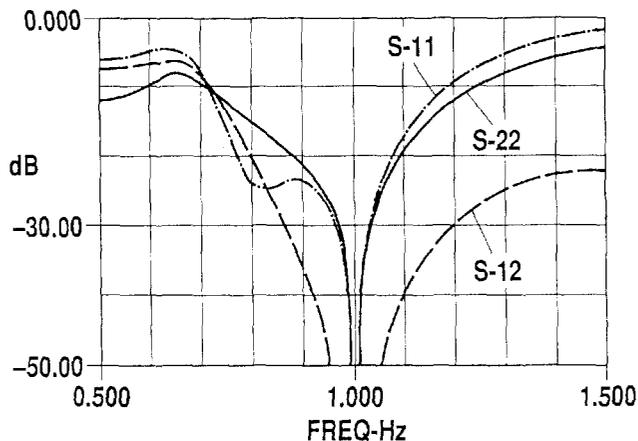


Figure 2d. Calculated isolation for the lumped circuit.

FREQ(MHz)	C(pF)	L(nH)
80	28.0	140
8000	0.28	1.40

Table I. The L and C values for lumped equivalent rat races at 80 and 8000 MHz.

and 8 GHz. Neglecting parasitics, only frequency scaling of the component values is required to shift the design center frequency. For example, a lumped element design at 80 MHz can be built that occupies no more than 40 square millimeters.

The result . . . is that the impedance of the elements in the low pass network is numerically equal to the impedance of the quarter wave line section being replaced.

Simplification of the above circuit, i.e. fewer components, at the expense of narrower bandwidth can be realized by employing a high-pass "pi" network instead of a "tee" network for the 270 degree transmission line segment. The resulting shunt inductors of the high-pass "pi" network have the same reactance, hence they are resonant with the shunt capacitors of the low-pass "pi" network at the design center frequency. Therefore they can be removed without affecting center frequency performance.

The resulting reduced circuit and its performance was evaluated but performance calculations revealed that its bandwidth over which performance was judged satisfactory was too reduced by this procedure and so the approach was not pursued.

The photograph of a MMIC lumped rat race hybrid designed for operation at 7.95 GHz is shown in Figure 3, the physical realization of the hybrid geometry of Figure 2. The coplanar waveguide probes and circuit pads are evident in the photograph. The hybrid itself occupies an area of only 1 square millimeter.

Through the cooperation of the Rome Air Development Center/EE this design this design was included on one of their GaAs hybrid circuit "foundry" runs. However, due to the time constraints that this entailed, it was not possible to optimize the circuit's analytical model with respect to the parasitics created by the interconnection of the L and C elements.

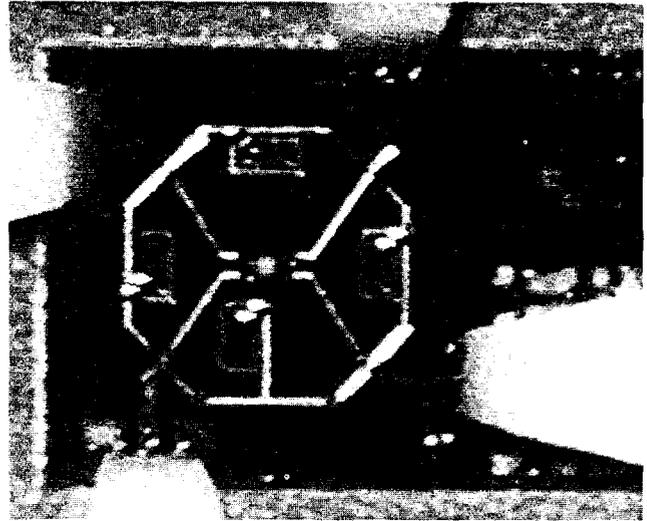


Figure 3. Photograph of a MMIC lumped rat race hybrid designed for operation at 7.95 GHz.

Specifically, in the initial design of the lumped hybrid, the lengths of the interconnecting transmission lines were assumed to have a length less than that realized in the final circuit. The effective additional inductance so created shifted the final center frequency downward from that of the intended design. The measured results, along with computed results using an analytical model reflecting the actual parasitics are shown in Figure 4. The realized performance is adequate for many applications.

spiral inductors . . . exhibit relatively high . . . losses consequently their use should be minimized

However, given the opportunity for further iterations, using a more accurate model for the parasitics, it should be possible both to design directly for the intended center frequency as well as to achieve a broader band over which acceptable performance is realized.

Summary

This technique of simulating line lengths with lumped element high and low pass filter sections yields electrical performance comparable to that of the transmission line circuit, but with significantly smaller area. The physical realization of the 8 GHz hybrid circuit demonstrates that calculated per-

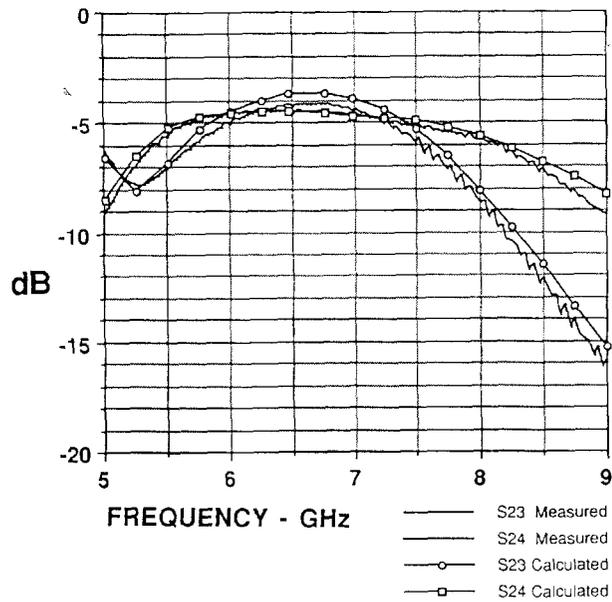


Figure 4a. Coupling of the Lumped Rat Race.

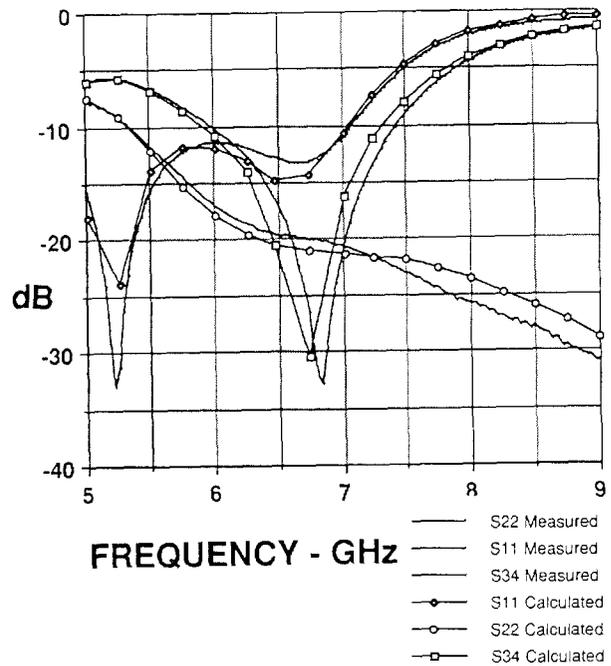


Figure 4c. Return Loss.

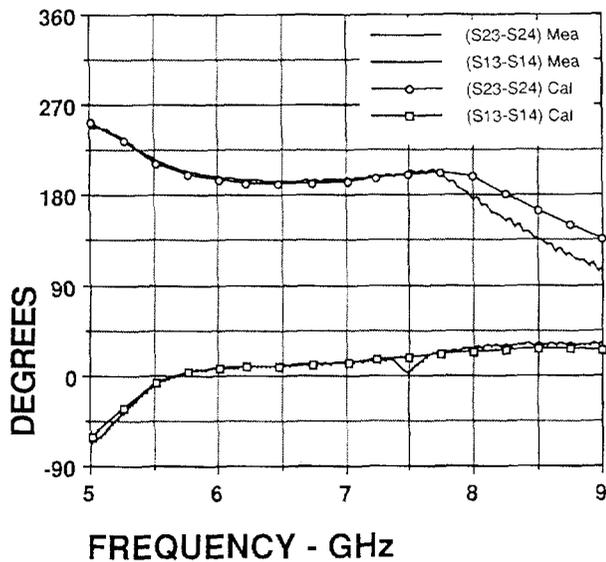


Figure 4b. Phase Response.

formance is attainable when parasitic effects of the interconnections are taken into account.

This technique is applicable at all frequencies in cases in which single and multiple quarter wavelength, distributed line lengths are required but would be too large for practical realization. This occurs in numerous distributed components such as quarter wave transformers, the 90 degree branch line hybrid and the Wilkinson power divider.

Acknowledgment

I would like to acknowledge the support of Gary Scalzi and Capt. William Cowan of RADC/EE for the manufacture of the hybrid circuit as well as Jim Devine and Stephanie Liberacki of MITRE for the RF tests that were performed on the circuit and assistance in the paper's preparation.

Appendix

The ABCD matrix for a lossless transmission line is

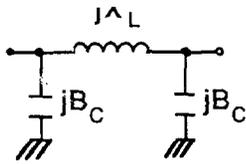
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\beta L & jZ_0 \sin\beta L \\ jY_0 \sin\beta L & \cos\beta L \end{bmatrix}$$

Lossless transmission line ($\alpha = 0$)

For $\beta L = 90^\circ$,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & jZ_0 \\ jY_0 & 0 \end{bmatrix}$$

This transmission line segment can be modeled as a low-pass "pi" network as shown below



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jB_C & 1 \end{bmatrix} \begin{bmatrix} 1 & jX_L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ jB_C & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 - X_L B_C & jX_L \\ jB_C(2 - X_L B_C) & 1 - X_L B_C \end{bmatrix}$$

Equating the matrix elements of the "pi" network to the matrix elements of the transmission line segment yields the following results

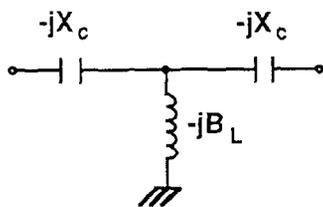
$$X_L B_C = 1 \text{ and } X_L = Z_0$$

$$\text{Therefore, } B_C = Y_0$$

The ABCD matrix for a 270 degree transmission line is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & -jZ_0 \\ -jY_0 & 0 \end{bmatrix}$$

This transmission line segment can be modeled as a high-pass "tee". The high-pass "tee" and its ABCD matrix are shown below



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & -jX_C \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -jB_L & 1 \end{bmatrix} \begin{bmatrix} 1 & -jX_C \\ 0 & 1 \end{bmatrix} =$$

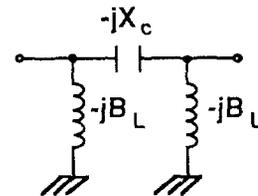
$$\begin{bmatrix} 1 - B_L X_C & -jX_C(2 - B_L X_C) \\ -jB_L & 1 - B_L X_C \end{bmatrix}$$

Equating the matrix elements of the "tee" network to the matrix elements of the transmission line segment yields the following results

$$B_L X_C = 1 \text{ and } B_L = Y_0$$

$$\text{Therefore, } X_C = Z_0$$

The narrowband version employs a high-pass "pi" network instead of a "tee" network for the 270 degree transmission line segment. This is shown below along with its ABCD matrix



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - B_L X_C & -jX_C \\ -jB_L & 1 - B_L X_C \end{bmatrix}$$

References

1. J. F. White, *Microwave Semiconductor Engineering*, Van Nostrand Reinhold, New York, 1982.
2. S. J. Parisi, Portions of this paper were presented at the 1989 MTT-S International Symposium in a paper entitled "180 Degree Lumped Element Hybrid."

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