# Measuring Complex Permittivity of Materials for Frequencies Under 18 GHz

By Israel Garcia-Ruiz and Carlos David Aviles-Castro, Centro Nacional de Metrologia; and Hildeberto Jardon-Aguilar, CINVESTAV del IPN

his article describes two systems for the measurement of complex permittivity of solids and other materials for the 100 MHz to 18 GHz frequency range. One system is based on a coaxial probe and the other on the measurement of reflected and transmitted waves with antennas. The dielectric characteristics of Teflon® PTFE and Revolite 4422 were measured and compared with both systems, and a good agreement was found. As will be shown, the open-ended coaxial technique is also appropriate for testing low-loss solids; the free-space technique, mainly used for millimeter and quasi-optical systems, can also be applied for the lower part of the microwave range. Implantation details, as well as material characterization results, are provided.

### Introduction

Complex permittivity is a crucial parameter in many RF and microwave applications [1] for making reasonably accurate wide-band and quick measurements. Several techniques have been developed for measuring complex permittivity [1-3]; however, some, such as resonant cavity or wave-guide transmission line cells, require test hardware machining and meticulous sample preparation. In addition, these techniques are destructive and have a limited frequency range.

Open-ended coaxial probe and free-space techniques have been used to characterize dielectric properties of materials [4-7]. The coaxial probe has proved to be more suitable for materials such as liquids, soft solids, powders and tissues, because good contact at the interface is easier to achieve. On the other hand, the free-space measurement system using horn

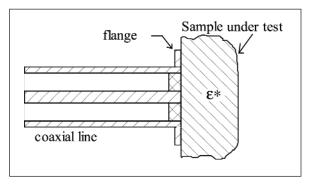


Figure 1. The open-ended coaxial probe.

antennas is a non-contact implementation that has found applications for characterization of materials such as low loss plastics, microwave absorbers and substrates.

Traditionally, several approaches have been used to calculate permittivity from reflection coefficient measurements using the coaxial probe: full-wave analysis [4-7], lumped circuit modeling [8] and antenna modeling [8] of the probe. The open-ended coaxial probe technique, however, is not definitive because it depends on the use of reference materials to calibrate the measurement system or ensure the solution convergence. The free-space method is more exact because it depends only on the use of the electromagnetic theory equations and on how closely the experimental system realization reproduces the conditions stated by the equations and the accuracy of reflection and transmission coefficients measurement.

# The open-ended coaxial probe technique

The main advantage of coaxial probes is their wide-band frequency response. When applying

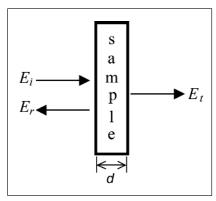


Figure 2. Diagram showing incident, reflected and transmitted waves.

the open-ended coaxial probe approach, good contact at the interface of the probe with the material under test is necessary. Therefore, coaxial probes may have been used more extensively for liquids, such as alcohol, water, saline solutions and oil, and soft solid materials, including organic tissues such

as human skin, as well as vegetables and powders, where such a good contact can be assured. In the case of solid materials, an air gap at the interface often introduces measurement errors. Several works have been devoted to evaluate the extent of these errors, and results can be found in literature. It was shown in [9] that an average surface roughness of 1  $\mu m$  produces an error of about 1 percent in the dielectric constant value of Teflon, with respect to the value that would correspond to zero roughness. Then, good contact and a smooth surface have to be assured.

A schematic for a coaxial open-ended probe is shown in Figure 1. The material being tested is placed in close contact with the probe's flat end; the reflection coefficient, measured with a vector network analyzer (VNA) or some other reflectometry technique, is related to the complex permittivity of the material, as shown in Equation (1).

$$\Gamma^*\left(\omega,\varepsilon^*\right) = \frac{Z_S - Z_0}{Z_S + Z_0} \tag{1}$$

where  $\omega$  is the angular frequency,  $Z_S$  is the probe impedance and  $Z_0$  is the characteristic impedance.

To calculate  $\varepsilon^*$  from the impedance measurements or the reflection coefficient, the bilinear calibration model, proposed first by Cole [10], is used. This model, an extension of the bilinear transformation used in microwave circuit theory [11], can also be seen as an equivalent circuit model incorporating a two port error correction box. Using this model,  $\varepsilon^*$  can be expressed by Equation (2):

$$\varepsilon^* = \frac{A\rho + \varepsilon^*_{ref}}{1 - B\rho} \tag{2}$$

where

$$\rho = \frac{\Gamma_{ref} - \Gamma_x}{\Gamma_{ref} + \Gamma_x}$$

and A and B are calibration constants that are calculated using two reference materials,  $\varepsilon^*_{ref}$  and  $\Gamma_{ref}$  are the permittivity and reflection coefficient of a known reference material; and  $\Gamma_x$  and  $\varepsilon^*$  are the reflection coefficient and permittivity of the material being tested.

The bilinear calibration model is very convenient, equations are linear and simple to solve and reasonably accurate results can be obtained if several considerations are taken into account. Suitable reference materials to solve Equation (2) have to be chosen, their permittivity values have to be well known and evenly distributed around expected values, calibration has to be carefully performed and good contact has to be assured at the probe interface.

# The free-space technique

When a plane wave is striking a slab of material of thickness d, as shown in Figure 2, the wave will be partially reflected back and partially transmitted across the material. Similar to wave-guided systems, reflected and transmitted fractions can be characterized by means of a reflection and a transmission coefficient.

The reflection coefficient can be written as (see Figure 2):

$$\Gamma = \frac{E_r}{E_i}$$

while the transmission coefficient can be written as:

$$\tau = \frac{E_t}{E_i}$$

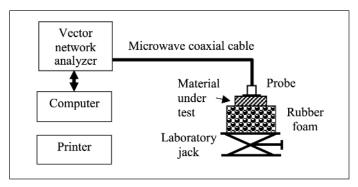
where  $E_i$ ,  $E_r$  and  $E_t$  represent the amplitudes of electric fields of incident, reflected and transmitted waves, respectively. The ratios expressed depend on the dielectric properties of the material. Their relationship can be expressed by using Equations (3) and (4) [11, 13]:

$$\Gamma = \frac{1 - \sqrt{\varepsilon^*}}{1 + \sqrt{\varepsilon^*}} \tag{3}$$

$$\tau = e^{-\gamma d} \tag{4}$$

where  $\gamma$  is the complex propagation constant. It is defined as

$$\gamma = \alpha + j\beta = j\frac{2\pi}{\lambda_0}\sqrt{\varepsilon^*}$$
 (5)



▲ Figure 3. Permittivity measurement system based on a coaxial probe.

Dielectric information can be extracted either from Equation (3), Equation (4), or both. The transmission coefficient is also a complex quantity that can be expressed in polar format equations, as shown in Equation (6):

$$\tau = Te^{j\theta} = e^{-\gamma d} \tag{6}$$

where *T* is the amplitude and  $\theta$  is the phase angle.

Applying the natural logarithm in both sides of Equation (6), we obtain:

$$\gamma = -\frac{1}{d}ln(T) - j\frac{l}{d}(\theta + 2\pi k) = \alpha + j\beta \tag{7}$$

where  $k=0, \pm 1, \pm 2, ...$ 

Equation (7) allows us to obtain the propagation constant from measurements of the complex transmission coefficient. The term on the right of Equation (5) can be divided into real and imaginary parts, as shown by Equations (8) and (9). Then, we can solve for  $\varepsilon$ ' and  $\varepsilon$ '', as shown by Equations (10) and (11).

$$\alpha = \frac{2\pi}{\lambda_0} \left\{ \frac{\varepsilon'}{2} \left[ \left( 1 + \frac{\varepsilon''^2}{\varepsilon'^2} \right)^{1/2} - 1 \right] \right\}^{1/2}$$
 (8)

$$\beta = \frac{2\pi}{\lambda_0} \left\{ \frac{\varepsilon'}{2} \left[ \left( 1 + \frac{\varepsilon''^2}{\varepsilon'^2} \right)^{1/2} + 1 \right] \right\}^{1/2}$$
 (9)

$$\varepsilon' = \left(\frac{\lambda_0}{2\pi}\right)^2 \left(\alpha^2 + \beta^2\right) \tag{10}$$

$$\varepsilon'' = \varepsilon' \left\{ \left[ \frac{2}{\varepsilon'} \left( \frac{\lambda_0 \alpha}{2\pi} \right)^2 + 1 \right]^2 - 1 \right\}^{1/2}$$
 (11)

By measuring the transmission coefficient  $\tau$ , calculating propagation constant  $\gamma$  from Equation (7), and substituting in Equations (10) and (11), the complex permittivity  $\varepsilon^*$  is obtained.

# **Experimental realization of measurement systems**

## a) Coaxial probe

Several probes were manufactured using inexpensive type-N and SMA coaxial connectors and adapters. These were modified by making a cut at one of the ends to get a flat interface. The best results were obtained with SMA, which showed good frequency response and excellent repeatability. Type SMA probes made from gold plated chassis connectors are quite small; this is an advantage, for example, when testing curved samples, because contact areas appear as approximate flat.

A coaxial probe-based system is shown in Figure 3. For liquids and pulverized materials, good contact is easy to achieve by immersing the probe in the sample container; however, for the case of solid materials, a piece of rubber foam and a laboratory jack were used to improve the contact. The sample being tested is sandwiched between the probe interface and rubber foam by adjusting the jack height. The probe is connected to the test port of an 8510C vector network analyzer through a precision microwave coaxial cable.

### b) Free-space

The free-space system, based on microwave transmission and reflection of waves, was set up with sets of WR-90 and WR-62 pyramidal standard gain horn antennas positioned on a rail system. VSWR of each set of antennas was measured to evaluate the extent of their frequency coverage and the similarity in their response. We found that the usable frequency range could be extended from 7 to 19 GHz using both sets.

The sample being tested was then placed between the antennas, with a fixture to hold the material sample. The distance between the antennas and the sample is set at minimum to fulfill the far field condition for the frequency range of operation of antennas:

Distance 
$$\geq \frac{2d^2}{\lambda}$$

where  $\lambda$  is the wavelength and d is the largest linear dimension of the antennas.

This ensures minimal coupling between antennas and provides a nearly flat wave front. The size of the samples sheet should be large enough to resemble a nearly infinite sample. In this case,  $60\times 60$  cm was chosen. Antennas are connected to the test ports of a vector analyzer through precision microwave cables, as shown in Figure 4.

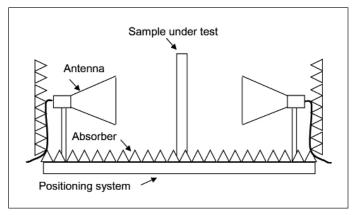
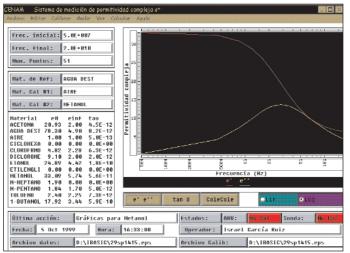


Figure 4. Permittivity measurement based on standard gain horn antennas.



lacktriangle Figure 5. Measurement program screen for  $\epsilon^*$ .

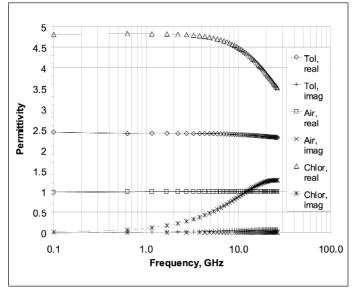


Figure 6. Real and imaginary parts of permittivity for chosen reference materials: toluene (tol), air and chloroform (chlor.).

To automate data capturing and processing of both permittivity measurement systems, a software program was written in BASIC; the measurement program screen is shown in Figure 5.

# Measurement calibration and material characterization results

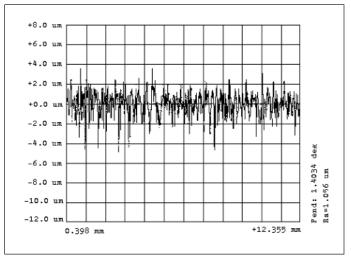
The coaxial probe measurement system has to be calibrated using reference standards as required by Equation (2). Chloroform, air and toluene were chosen as suitable references. Their permittivities are known, all three are available with high purity, are easy to remove (which avoids probe contamination) and have permittivities of the same order as the expected values for those materials to be tested. Thus, the measured permittivity accuracy is improved, especially for the imaginary part, which is very small in low-loss materials. Permittivities for these reference materials are shown in Figure 6. These plots were generated using the Debye dielectric relaxation model [1], and the permittivity values to solve the equation of the model were taken from [1, 14]. A permittivity of  $\varepsilon^* = 1 + j0$  has been assigned to air.

After measuring the reference materials and calculating the calibration constants A and B of Equation (6), error correction should be done at the probe interface. No prior or further calibration is needed. Tests performed by running a prior VNA traditional calibration showed no improvement in the final calculated permittivity values.

Preparation of solid samples for testing requires only a flat surface for mating with the probe interface. Samples of Teflon PTFE and Rexolite 4422 were prepared in  $2\times2\times1$  cm dices, and a flat surface of  $2\times2$  cm was polished. Surface finish tests, performed by a dimensional metrology laboratory, showed typical average roughness of  $R_a=1~\mu\mathrm{m}$  (see Figure 7).

Preliminary tests performed on the Teflon PTFE samples showed that a nearly flat dielectric response was obtained for the 500 MHz to 20 GHz range, with a mean value of  $\varepsilon'=2.0$  and the loss factor increasing with frequency. The 2.0 value represents a 5 percent difference compared to the well-known value of 2.1. We identified that the source of this difference could be incorrect permittivity values in one or more reference materials and/or inadequate contact at probe interface. After some testing, we found that the calibration routine needed to be modified and a solid reference material needed to be included. Although almost perfect contact is achieved with liquids, a solid surface can present imperfections, such as roughness and waviness, which have to be taken into account.

Toluene was replaced as reference material by a solid. When making tests on Rexolite we used Teflon ( $\varepsilon' = 2.1$ ) as reference. For testing Teflon, we used Rexolite ( $\varepsilon' = 2.53$ ) as reference. After performing the new calibration



▲ Figure 7. Typical roughness profile of samples under test.

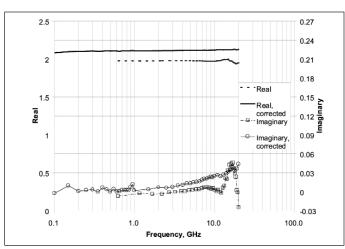
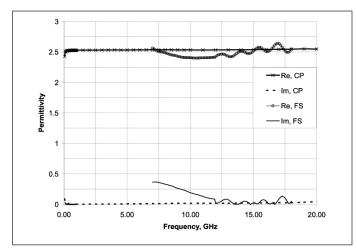


Figure 8. Permittivity of Teflon® PTFE, measured with the coaxial probe system.



▲ Figure 9. Permittivity of Rexolite 4422 measured with coaxial probe and free-space systems, CP: coaxial probe, FS: free-space.

routine, samples were measured again and the results represented a substantial improvement. Figure 8 shows real and imaginary parts of permittivity of Teflon PTFE for both conditions; the dielectric response is even smoother, and the value for PTFE is closer to 2.1.

Similar results were obtained for Rexolite 4422. We also measured this material using the free-space system, which does not require the use of any reference material. Figure 9 shows permittivity values obtained with both systems, coaxial probe and free-space. Measured values closely match those reported, as obtained through other techniques [1]. The coaxial probe system allows the entire response to be obtained in a single run, with typical measurement times of about one minute. In the free-space system, it is necessary to switch the sets of antennas and recalibrate the system, which does not require the use of reference materials. Accuracy depends on the calibration of the vector network analyzer, the free-space propagation condition approximation and the dimensional accuracy of sample and positioning system.

### **Conclusions**

Two systems for measuring complex permittivity in the microwave frequency range were developed, tested and compared. A system based on a coaxial probe allows obtaining the dielectric properties of materials in a wide band very quickly, though it depends on the use of reference materials with known permittivities for system calibration. A free-space system, based on the use of standard gain pyramidal horn antennas, is an absolute measurement system that does not require the use of any reference material; it also allows comparing results obtained using other measurement methods and covers the range from 8 to 18 GHz. Several materials were tested; dielectric characteristics for Teflon PTFE and Revolite 4422 were obtained and compared. A good agreement between results obtained from both measurements was shown.

Coaxial probe has been widely used mostly for testing liquids and powders; however, this technique is also suitable for testing solid materials. Good contact at the probe interface is provided. Probe calibration is also important for accurate results and suitable reference materials have to be chosen.

For example, the permittivity of the reference materials should be of the same order as those for the expected values for materials under test; this consideration improves measurement of imaginary part of permittivity, which is near zero for low loss materials. Errors introduced by imperfect contact of the probe with the material under test were corrected to some degree by using one solid material in the set of references.

The free-space system allows obtaining the complex permittivity of sheets of materials for the 8 to 18 GHz. For this system, no reference materials of known permittivity are needed, there is no contact between sample

under test and measurement system and waves penetrate the whole material, providing a bulk permittivity evaluation. Comparison of results for Rexolite 4422 samples obtained with both systems shows that there is a good agreement between them, as well as with those obtained using other techniques.

# **Acknowledgment**

The authors wish to thank Consejo Nacional de Ciencia y Tecnologia (CONACyT) for supporting this research.

### References

- 1. Arthur Von Hippel, *Dielectric Materials and Applications*, Boston: Artech House, 1995.
- 2. H.E. Bussey, "Measurement of RF Properties of Materials, A Survey," *Proceedings of the IEEE*, Vol. 55, June 1967: 1046-1053.
- 3. K. Fenske and D. Misra, "Dielectric Materials at Microwave Frequencies," *Applied Microwave and Wireless*, Vol. 12, No. 10, October 2000: 92-100.
- 4. J. Baker-Jarvis, M.D. Janezic, P.D. Domich and R.G. Geyer, "Analysis of an Open-Ended Coaxial Probe with Lift-off for Non-destructive Testing," *IEEE Transactions on Instruments & Measurements*, Vol 43, No. 5, October 1994: 711-718.
- 5. S. Jenkins, T.E. Hodgett, R.N. Clarke, A.W. Preece, "Dielectric Measurements on Reference Liquids using Automatic Network Analyzers and Calculable Geometries," *Measurement Science & Technology*, Vol. 1, 1990: 691-702.
- 6. C. Lin and K. Chen, "Determination of Electromagnetic Properties of Materials Using Flanged Openended Coaxial Probe Full Wave Analysis," *IEEE Transactions on Instruments & Measurements*, Vol. 44, 1995: 19-27.
- 7. D. Misra, "On the Measurement of the Complex Permittivity of Materials by an Open-ended Coaxial Probe," *IEEE Microwave and Guided Wave Letters*, Vol. 5, 1995: 161-163.
- 8. D.Berube and F. Gannouchi. "A Comparative Study of Four Open-ended Probe Models for Permittivity Measurements of Lossy Dielectric/-Biological Materials at Microwave Frequencies," *IEEE Transactions on MTT*, Vol. 44, 1996: 1928-1934.
- 9. M. Arai, J.G.P. Binner and T.E. Cross, "Estimating Errors Due to Sample Surface Roughness in Microwave Complex Permittivity Measurements Obtained Using a Coaxial Probe," *Electron Letters*, Vol. 31, 1995: 115-117.
- 10. R.H. Cole, J.G. Berberian, S. Mashimo, G. Chryssikos and A. Burns, "Time Domain Reflection Methods for Dielectric Measurements to 10 GHz," *Journal of Applied Physics*, Vol. 66, July 1989: 793-802.
  - 11. D.K. Ghodgaonkar, V.V. Varadan and V.K. Varadan.

- "Free-space Measurement of Complex Permittivity and Complex Permeability of Magnetic Materials at Microwave Frequencies," *IEEE Transactions on Instruments & Measurements*, Vol. 39, 1990: 387-394.
- 12. G.F. Engen. "Microwave Circuit Theory and Foundations of Microwave Metrology," *IEE Electrical Measurements*, Series 9, Peter Peregrinus Ltd., England, 1992.
- 13. S. Trabelsi, A. Kraszewsky and S. O. Nelson. "Nondestructive Microwave Characterization for Determining the Bulk Density and Moisture Content of Shelled Corn," *Measurement Science & Technology*, Vol. 9, 1998: 1548-1556.
- 14. K. Folger, T. Friss, J. Hilland and T. Tjomsland, "A Broadband and High Sensitivity Dielectric Spectroscopy Measurement System for Quality Determination of Low-Permittivity Fluids," *Measurement Science & Technology*, Vol. 6, 1995: 995-1008.

## **Author Information**

Israel Garcia-Ruiz received his master's degree from the Research and Advanced Studies Center of IPN in Mexico City, Mexico. He is currently pursuing his Ph.D. degree, working on techniques for material characterization using RF and microwave energy. He is employed as a microwave metrologist with CENAM, the National Measurements Institute of Mexico, where he has collaborated on the establishment of the high frequency measurement capabilities for the past eight years. He has coauthored several technical papers, as well as presentations for conferences. He may be reached by E-mail: igarcia@cenam.mx; Tel: + 52-4-2110500; or Fax: + 52-4-2110548.

Carlos David Aviles-Castro received his Ph.D. in instrumentation and measurements from the Bordeaux I University in France. Since 1992, he has been employed at CENAM; from 1992 to 1999, he headed the Electromagnetic Measurements Division; and since 1999, he has served as the research project leader in the same division. He is currently working on high accuracy electrical measurements, particularly in the Josephson effect, permittivity measurements and moisture measurements based on microwaves. He may be reached by E-mail: caviles@cenam.mx; Tel: + 52-4-2110541; or Fax: + 52-4-2110548.

Hildeberto Jardon-Aguilar received a bachelor's degree from the ESIME of the Polytechnic Institute of Mexico and a Ph.D. from Moscow Technical University of Communications and Informatics. He is employed as a full-time professor at the Research and Advanced Studies Center of IPN. He is author of more than 100 technical papers, as well as four books. He may be reached by E-mail: hjardon@enigma.red.cinvestav.mx; Tel:+52-5-7473779; or Fax: +52-5-7473880.