

Cascaded Efficiency of Power Amplifiers

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Radio frequency (RF) power amplifier (PA) designers have a choice of many device technologies, such as a metal semiconductor field effect transistor (MES-FET), metal-oxide semiconductor field-effect transistor (MOSFET), heterojunction bipolar transistor (HBT) and high-electron-mobility transistor (HEMT). Designers can use the strengths of a particular technology to meet performance or cost requirements.

With short product cycles and many trade-offs to consider, it is important to streamline the design process as much as possible. Circuit and device level simulations can model the behavior of single and cascaded amplifier stages with any desired amount of detail. However, it speeds up the design process to begin with basic equations to approximate circuit performance and then add more detail when appropriate.

Textbook equations for gain, noise figure and third-order intercept point of cascaded amplifiers are familiar to designers. These equations can be plugged into a spreadsheet to determine performance of a cascade of gain blocks. Such tools are used by systems designers to evaluate the effects of losses and early compression on the performance of a transmit or receive chain. They are also helpful to multistage amplifier designers, especially those working on linear amplifiers, since each stage has an effect on the overall performance of the chain and trouble spots can be identified.

Cascaded efficiency is a parameter often missing from spreadsheet calculations, but its importance cannot be understated. In linear amplifiers, there is a direct trade off between the amount of backoff from saturation to achieve linearity and the efficiency. It is a frus-

trating experience for designers to watch the efficiency they fought so hard to achieve disappearing as the linearity of the amplifier is brought within specification.

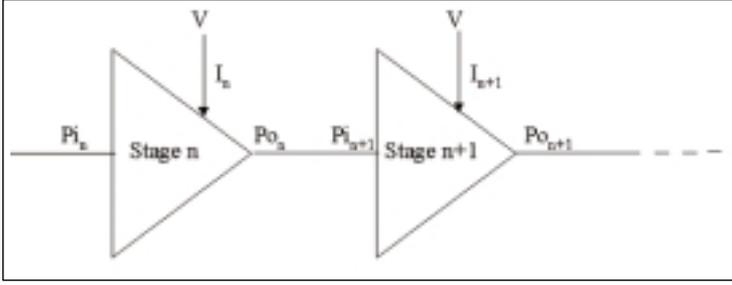
The main source of PA efficiency, the output stage, is by far the largest consumer of current and has the highest output power in the chain. It also generates most of the linearity problems, since it must run as close as possible to compression to achieve the required efficiency.

Therefore, previous stages are backed off as much as possible from saturation to minimize their role in the nonlinear behavior of the amplifier. This results in the previous stages being less efficient than the output stage, but their contribution to the overall efficiency is smaller because of their smaller output power. How much these small, inefficient driver stages bring down the overall efficiency is not obvious without knowledge of the cascaded efficiency.

To this end, a simple equation for the efficiency of cascaded amplifiers is derived. It can be used in a spreadsheet by specifying the gain and efficiency of each stage. With this equation, it is possible to balance the trade-off between linearity and efficiency in the driver stages so that they are not designed to be over-efficient. In addition, a similar equation gives an estimate of the driver stage efficiency needed to hold the drop in overall efficiency to a specified amount when the stages are cascaded.

Amplifier efficiency

Efficiency measures how well an amplifier converts direct current (DC) power into RF power. When efficiency is low, the amplifier consumes excessive DC power, causing overheating, shortened device life and quick battery drain.



▲ Figure 1. General section of amplifier cascade.

Efficiency is measured in two ways: terminal (drain or collector) efficiency and power-added efficiency (PAE).

Terminal efficiency is given by:

$$TE = \frac{P_{out}}{P_{dc}} \quad (1)$$

which is the ratio of RMS output power to the DC power consumed. PAE is given by [1]:

$$PAE = \frac{P_{out}}{P_{dc}} \cdot \left(1 - \frac{1}{G}\right) \quad (2)$$

where G is the gain of the amplifier expressed as a ratio.

It is assumed that the input and output matching network losses are included in the gain. The gain term accounts for the fraction of input power that appears at the output due to the amplifier's finite gain. PAE can be thought of as a "conversion efficiency," measuring the actual conversion of DC power to RF power by the active devices while accounting for the fraction of input power contributing to the output. PAE measures the intrinsic performance of the amplifying devices and is most often used in amplifier design. When gain is very high, the second term in parentheses becomes negligible and the PAE is the same as the terminal efficiency. Efficiency is most often expressed as a percentage.

Derivation of the cascaded efficiency equation

Figure 1 shows a section of an amplifier chain used in the derivation of the cascaded efficiency equation. Stage n is connected to stage $n+1$ in cascade. Stage n has gain (G_n), current (I_n), input power (P_{i_n}) and output power (P_{o_n}). Stage $n+1$ has the same variables, but with the $n+1$ subscript. The amplifier chain continues to the right and left, and all amplifiers are fed from the same voltage source V . There are N amplifiers in the chain, and the subscript n goes from 1 to N .

Since DC power consumed by an active device is the product of voltage and current, Equation (2) may be written as (including the conversion to percentage and replacing P_{out} with P_o):

$$PAE = \frac{P_o}{V \cdot I} \cdot \left(1 - \frac{1}{G}\right) \cdot 100 \quad (3)$$

For cascaded amplifiers, the total gain will be the product of the individual gains; the total current will be the sum of the individual currents:

$$PAE_{tot} = \frac{P_o}{V \cdot \left(\sum_{i=1}^N I_i\right)} \cdot \left[1 - \frac{1}{\prod_{n=1}^N G_n}\right] \cdot 100 \quad (4)$$

This would be all that was necessary if the current of each stage were an independent variable. However, it is more useful if the efficiency of each stage is used as an independent variable, since this is a design parameter. From Equation (3), we solve for the current of stage i as:

$$I_i = 100 \cdot P_{oi} \cdot \frac{(G_i - 1)}{(PAE_i \cdot V \cdot G_i)} \quad (5)$$

The output power of stage i , P_{oi} , is equal to the output power of the entire amplifier chain divided by the product of the gains of all the following stages:

$$P_{oi} = \frac{P_o}{\begin{cases} \prod_{k=i+1}^N G_k & \text{if } i < N \\ 1 & \text{otherwise} \end{cases}} \quad (6)$$

where the vertical bar notation signifies a test for the value of i , since the index k is not valid for $i = N$. When $i = N$, the value of 1 is substituted for the gain product signifying that the last output power of the chain is P_o .

Substituting Equation (6) and Equation (5) into Equation (4) results in:

$$PAE_{tot} = \frac{1}{\sum_{i=1}^N \frac{1}{\begin{cases} \prod_{k=i+1}^N G_k & \text{if } i < N \\ 1 & \text{otherwise} \end{cases}} \cdot \frac{(G_i - 1)}{(PAE_i \cdot G_i)}} \cdot X \cdot \left[1 - \frac{1}{\prod_{n=1}^N G_n}\right] \quad (7)$$

The factor of 100, P_o and the voltage V have canceled out because they are now included as part of the PAE of

each stage. Equation (7) is now a function only of the gains and efficiencies of each stage.

Equation for final stage and driver

In practice, the efficiency of an amplifier is dominated by the output stage and driver because each successive stage toward the input consumes less DC power and adds gain to the cascade, thereby contributing less and less to the reduction of overall efficiency and further reducing the contribution of input power to the output. A version of Equation (7) can be derived that includes the effect of two cascaded stages only and uses the percentage point degradation of PAE as a parameter.

Using the subscripts 1 and 2 to denote parameters of the first and second stages of the cascade, respectively, the efficiency of the output stage can be written by the equation:

$$PAE_2 = \frac{P_{o2}}{V \cdot I_2} \cdot \left(1 - \frac{1}{G_2}\right) \cdot 100 \quad (8)$$

Total efficiency is obtained by replacing I_2 with the sum of both currents and G_2 with the product of the gains, as in the general case.

$$PAE_{tot} = \frac{P_{o2}}{V \cdot (I_1 + I_2)} \cdot \left(1 - \frac{1}{G_1 \cdot G_2}\right) \cdot 100 \quad (9)$$

The percentage point difference in efficiency ΔPAE between an amplifier with only the final stage and one with both stages operating is:

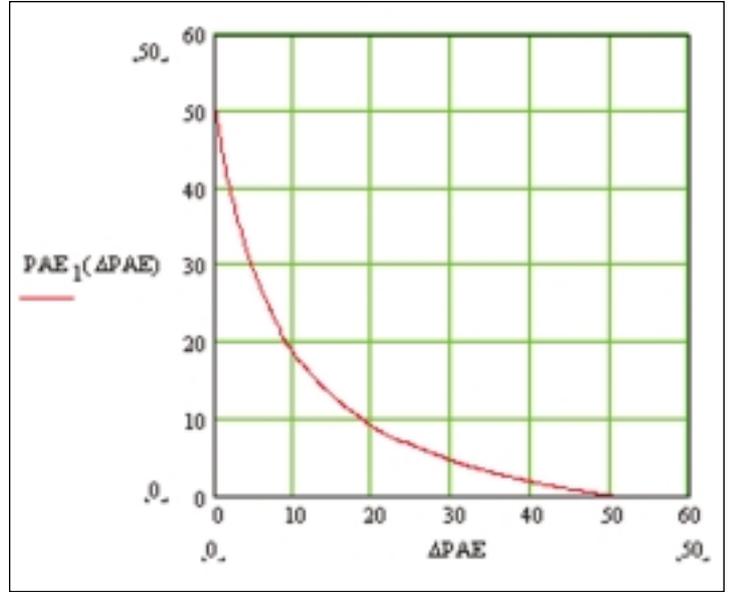
$$PAE_2 - PAE_{tot} = PAE_2 - \frac{P_{o2}}{V_{cc} \cdot (I_1 + I_2)} \cdot \left(1 - \frac{1}{G_1 \cdot G_2}\right) \cdot 100 \quad (10)$$

For example, an output stage with a efficiency of 50 percent coupled with a driver that brings the overall efficiency down to 45 percent would have a ΔPAE of 5. The efficiency of the driver stage is:

$$PAE_1 = \frac{P_{o1}}{V \cdot I_1} \cdot \left(1 - \frac{1}{G_1}\right) \cdot 100 \quad (11)$$

Solving Equation (11) for I_1 gives:

$$I_1 = 100 \cdot P_{o1} \cdot \frac{(G_1 - 1)}{(PAE_1 \cdot V \cdot G_1)} \quad (12)$$



▲ Figure 2. Graph of PAE_1 versus ΔPAE for the example.

Substituting Equation (12) into Equation (10) gives:

$$\Delta PAE = PAE_2 - \frac{100}{V} X \left[100 \cdot P_{o1} \cdot \frac{(G_1 - 1)}{(PAE_1 \cdot V \cdot G_1)} + I_2 \right] X \left[1 - \frac{1}{(G_1 \cdot G_2)} \right] \quad (13)$$

The preceding equation can also be solved for PAE_1 (see Equation (14) below).

The product of V and I_2 may be eliminated by solving for it in Equation (8):

$$V \cdot I_2 = \frac{P_{o2}}{PAE_2} \cdot \left(1 - \frac{1}{G_2}\right) \cdot 100 \quad (15)$$

Substituting Equation (15) into Equation (14) and replacing P_{o1} with P_o/G_2 results in:

$$PAE_1 = \frac{(PAE_2 - \Delta PAE) \cdot (G_1 - 1) \cdot PAE_2}{[\Delta PAE \cdot (G_2 - 1) + PAE_2] \cdot G_1 - PAE_2} \quad (16)$$

$$PAE_1 = \frac{100 \cdot G_2 \cdot P_{o1} \cdot (PAE_2 - \Delta PAE) \cdot (1 - G_1)}{G_1 \cdot G_2 \cdot [I_2 \cdot V \cdot (PAE_2 - \Delta PAE) - 100 \cdot P_{o2}] + 100 \cdot P_{o2}}$$

▲ Equation 14.

Equation (16) is a simple expression that gives the driver efficiency required to keep the reduction in efficiency of the output stage to a value of ΔPAE . Remember that the power gains G_1 and G_2 in Equation (16) are in ratio form. They can be converted from decibels using the formula:

$$G_{1,2} = 10^{\frac{G_{1,2,\text{dB}}}{10}} \quad (17)$$

PAE and ΔPAE must be entered as percentages.

Use of the efficiency equation in amplifier design

Suppose a power amplifier is required to produce +29 dBm linear RF power at an efficiency of 45 percent. An output stage is designed using load pull or simulation to give +29 dBm with a gain of 8 dB (6.31 in ratio). The best PAE available given the linearity constraints is 50 percent. From this information, ΔPAE is 5 (50 to 45 percent).

For an output power of 29 dBm, a driver stage would have to produce at least 21 dBm linear power, given the 8 dB gain of the output stage. If the gain G_1 of the driver stage is 10 dB, then from (16) the efficiency of the driver would need to be 28.3 percent for the overall efficiency of the two-stage amplifier to be 45 percent.

Figure 2 shows a graph of PAE_1 versus ΔPAE for this gain condition. When below 50 percent, the driver efficiency lowers the overall efficiency of the cascade. At 20 percent, a reasonable efficiency for a linear driver, the overall efficiency declines by about 10 points from that of the output stage alone. This illustrates why it is difficult to get good efficiency in a linear amplifier chain. Each of the stages must strike an optimal balance between linearity and efficiency unless there is significant phase cancellation of individual distortion products which would not be a reliable technique without external circuitry. Another possible solution is to employ a linearization technique so that the amplifier stages could be

operated closer to compression, thus improving their efficiency. ■

References

1. Stephen A. Mass, *Nonlinear Microwave Circuits*, New York: IEEE Press, 1997.

Author information

Howard Patterson received a bachelor of science degree in electrical engineering from the University of Kansas in 1982 and a master of science degree in electrical engineering from Arizona State University in 1989. He is employed by the Motorola Semiconductor Products Sector in Tempe, AZ, where he designs and develops linear power amplifier modules for cellular and personal communications system band handsets. His interests include modeling and simulation of PAs and passive components and advanced design techniques to optimize amplifier performance. He may be reached via E-mail: rzaw10@email.sps.mot.com; or Tel: 480-413-5537.