Unfiltered FQPSK: Another Interpretation and Further Enhancements

Part 2: Suboptimum reception and performance comparisons

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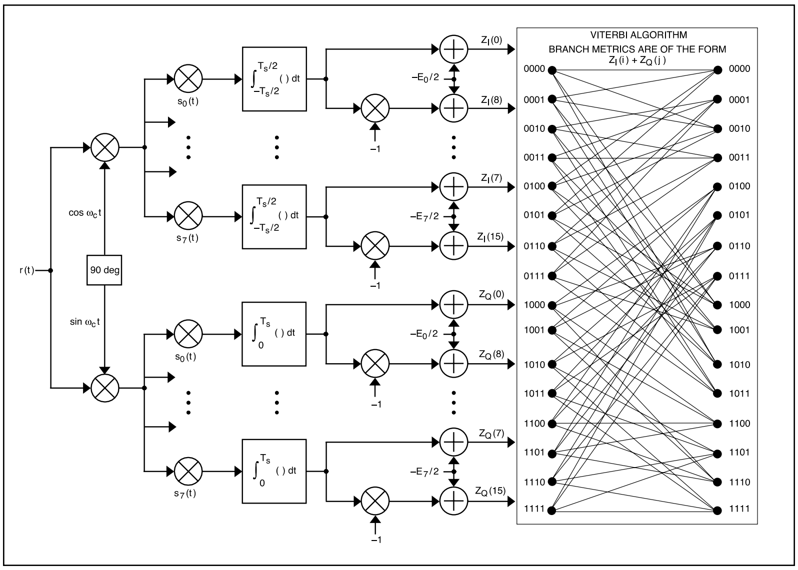
Previously, we discussed the transmitter implementation and optimum reception for a new interpretation of unfiltered Feher-Patented Quadrature Phase-Shift Keying (FQPSK). In the conclusion to this article, we will cover two suboptium receivers for symbol-by-symbol detection of FQPSK, including performance comparisons.

This is part two of a two-part article. Part one was published in the February 2000 issue of *Applied Microwave & Wireless*.



▲ **Figure 11. The optimum trellis-coded receiver for FQPSK.**

Suboptimum receivers

In accordance with our earlier discussion, we shall consider two suboptimum receivers for symbol-by-symbol detection of FQPSK, the difference being the manner in which the detector is matched to the received signal. For the average matched filter case, the detector is implemented as a multiplication of the received signal, followed by an I&D filter and binary hard decision device (Figure 11).

For the OQPSK receiver, the detector is purely an I&D (i.e., matched to a rectangular pulse). Thus, we can cover both cases at the same time leaving S as an arbitrary premultiplication pulse shape and later substitute the appropriate waveform.

Assuming the M-ary symbol-by-symbol representation of FQPSK just described, then the decision variable Z in Figure 12 is given by

$$Z = \int_0^{T_s} S(t)\overline{S}(t)dt + \int_0^{T_s} n(t)\overline{S}(t)dt \overset{\Delta}{=} Z + N \quad (28)$$

where $S(t)$ is the transmitted waveform in $0 \le t \le T_s$ and ranges over the set of eight waveforms in (23a) and (23b) with equal probability. The random variable N is zeromean Gaussian with variance $\sigma_N^2 = N_0 E_S/2$ where

$$E_{\overline{S}} \overset{\Delta}{=} \int_0^{T_s} \overline{S}^2(t)dt$$

Thus, the I channel symbol error probability (same as the Q channel symbol error probability) conditioned on the particular $S(t) = S_i(t)$

corresponding to the transmitted symbol $d_{I0} = 1$ is easily shown to be

$$P_{si}(E) = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{1}{N_0}\frac{\left(\int_0^{T_s} S_i(t)\bar{S}(t)dt\right)^2}{E_{\bar{S}}}}\right) \qquad (29)$$

and hence the average symbol error probability is

$$P_s(E) \overset{\Delta}{=} \frac{1}{8}\sum_{i=0}^{7} P_{si}(E) \qquad (30)$$

Conventional OQPSK receiver

For the conventional OQPSK receiver, we set $S(t) = 1$, or equivalently, $E_{\bar{S}} = T_s$ in (29), resulting in

$$P_{si}(E) = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{T_s}{N_0}\left(\frac{1}{T_s}\int_0^{T_s} S_i(t)dt\right)^2}\right) = \frac{1}{2}\text{erfc}\left(\sqrt{\left(\frac{32}{7+2A+15A^2}\right)\frac{\bar{E}_b}{N_0}\langle S_i(t)\rangle^2}\right) \qquad (31)$$

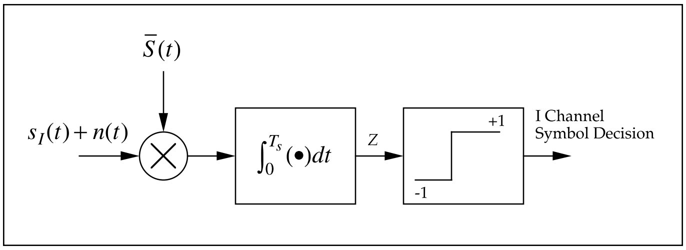
where

$$\langle S_i(t)\rangle = (1/T_s)\int_0^{T_s} S_i(t)dt$$

denotes the time average of $s_i(t)$. Evaluating these time averages from (23b), substituting each of them in (31) and then performing the average as in (30) gives the final desired result for average symbol error probability, namely,

$$P_{si}(E) = \frac{1}{2}\text{erfc}\left(\sqrt{\left(\frac{32A^2}{7+2A+15A^2}\right)\frac{\bar{E}}{N_0}}\right) + \frac{1}{8}\text{erfc}\left(\sqrt{\left(\frac{8A^2\left(1+\frac{2}{\pi}\right)^2}{7+2A+15A^2}\right)\frac{\bar{E}_o}{N_0}}\right) + \frac{1}{16}\text{erfc}\left(\sqrt{\left(\frac{8(1+A)^2}{7+2A+15A^2}\right)\frac{\bar{E}_o}{N_0}}\right) + \frac{1}{8}\text{erfc}\left(\sqrt{\left(\frac{2\left(1+\frac{4}{\pi}+A\right)^2}{7+2A+15A^2}\right)\frac{\bar{E}_o}{N_0}}\right)$$

$$+\frac{1}{16}\text{erfc}\left(\sqrt{\left(\frac{32A^2\left(\frac{2}{\pi}\right)^2}{7+2A+15A^2}\right)\frac{\bar{E}_o}{N_0}}\right) + \frac{1}{16}\text{erfc}\left(\sqrt{\left(\frac{32\left(\frac{2}{\pi}\right)^2}{7+sA+15A^2}\right)\frac{\bar{E}_o}{N_0}}\right) \qquad (32)$$

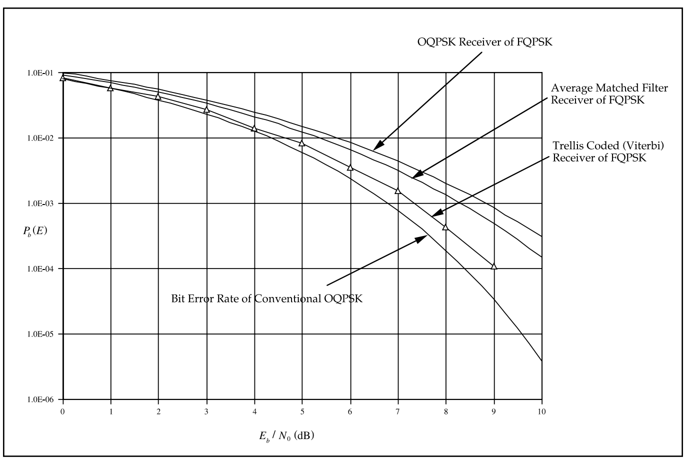


▲ **Figure 12. Suboptium receiver for FQPSK based on symbol-by-symbol detection.**

Average matched filter receiver

For the average matched filter, we need to compute the correlations of each of the pulse shapes in (23a) and (23b) with the average pulse shape $S(t)$ and also the energy $E_{\bar{S}}$ of the average pulse shape. Rewriting (29) in a form analogous to (31), namely,

$$P_{si}(E) = \frac{1}{2}erfc\left(\sqrt{\left(\frac{32}{7+2A+15A^2}\right)\frac{\bar{E}_o}{N_0}\frac{\left(\frac{1}{T_s}\int_0^{T} S_1(t)\bar{S}(t)dt\right)^2}{\frac{1}{T_s}E_{\bar{S}}}}\right) \qquad (33)$$



▲ **Figure 13. Bit error rate performance of various receivers of FQPSK modulation (reference curve is bit error of OQPSK).**

then the results necessary to evaluate (33) are tabulated as:

$$\frac{1}{T_s}\int_0^{T_s} S_0(t)\bar{S}(t)dt = \frac{A}{4}\left[\frac{1}{2}+\frac{2}{\pi}+A\left(\frac{3}{2}+\frac{2}{\pi}\right)\right]$$
$$\frac{1}{T_s}\int_0^{T_s} S_1(t)\bar{S}(t)dt = \frac{1}{T_s}\int_0^{T_s} S_4(t)\bar{S}(t) =$$
$$\frac{A}{4}\left[\frac{1}{2}+\frac{5}{3\pi}+A\left(1+\frac{7}{3\pi}\right)\right]$$
$$\frac{1}{T_s}\int_0^{T_s} S_2(t)\bar{S}(t)dt =$$
$$\frac{1}{4}\left[\frac{3}{8}+\frac{4}{3\pi}+A\left(\frac{3}{4}+\frac{2}{\pi}\right)+A^2\left(\frac{7}{8}+\frac{2}{3\pi}\right)\right] \tag{34}$$
$$\frac{1}{T_s}\int_0^{T_s} S_3(t)\bar{S}(t)dt = \frac{1}{T_s}\int_0^{T_s} S_6(t)\bar{S}(t)dt =$$
$$\frac{1}{4}\left[\frac{7}{16}+\frac{4}{3\pi}+A\left(\frac{5}{8}+\frac{7}{3\pi}\right)+A^2\left(\frac{7}{16}+\frac{1}{3\pi}\right)\right]$$
$$\frac{1}{T_s}\int_0^{T_s} S_5(t)\bar{S}(t)dt = \frac{A}{2}\left[\frac{1}{4}+\frac{2}{3\pi}+A\left(\frac{1}{4}+\frac{4}{3\pi}\right)\right]$$
$$\frac{1}{T_s}\int_0^{T_s} S_7(t)\bar{S}(t)dt = \frac{1}{2}\left[\frac{1}{4}+\frac{2}{3\pi}+A\left(\frac{1}{4}+\frac{4}{3\pi}\right)\right]$$

and

$$\frac{1}{T_s}E_{\bar{S}} = \frac{1}{16}\left[\begin{array}{l}(1+A)^2\left(\frac{3}{2}+\frac{4}{\pi}\right)+\frac{3}{8}(1-A)^2 \\ -2\left(1-A^2\right)\left(\frac{1}{2}+\frac{2}{3\pi}\right)\end{array}\right] \tag{35}$$

Finally, substituting (34) and (35) into (33) and averaging as in (30) gives the desired result, which we shall not explicitly write in closed form.

Average bit error probability performance

The average BEP of the two suboptimum receivers is illustrated in Figure 13 for the case $A = 1\sqrt{2}$, which is the usual value chosen for implementation of FQPSK. These results are obtained directly from equation (32) for the OQPSK receiver and from (30) in combination with (33) - (35) for the average matched filter receiver.

Also included in this figure is the performance corresponding to the optimum uncoded OQPSK receiver (same performance as for uncoded BPSK). This performance figure is given as:

$$P_b(E) = \frac{1}{2}\operatorname{erfc}\sqrt{E_b / N_D}$$

The figure also includes simulation results obtained for the optimum trellis coded receiver of Figure 11. We observe, as one might expect, that the average matched filter receiver outperforms the OQPSK receiver, since an attempt to match the transmitted pulse shape (even on an average basis) is better than no attempt at all.

We also observe that the trellis coded receiver at $P_b(E) = 10^{-4}$ is more than 1 dB better than the average matched filter receiver, granted that the latter is considerably simpler in implementation. Finally, for the same average BEP, the trellis coded receiver of FQPSK is only about 0.6 dB inferior to uncoded OQPSK performance, which is a relatively small penalty paid for the vast improvement in PSD afforded by the former relative to the latter. ■

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