Finite Element Analysis of Planar Couplers

A technical review and analysis of electromagnetic coupling between microstrip lines

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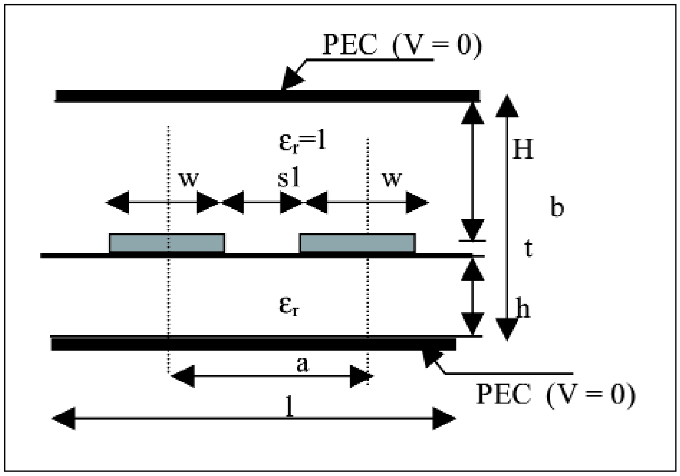
In this paper, we present the analysis of electromagnetic coupling between two lossy and inhomogeneous microstrip lines. This analysis is based on a numerical resolution of Laplace's equation by the finite element method (FEM), because this complex configuration of lines does not have rigorous analytical resolution. The modelling of this structure is designed to yield the primary parameters (L, C, R, G) of the equivalent electronic circuit and the secondary parameters described by the impedances (Z_{0e}, Z_{0o}) of the even and odd modes and the coupling coefficient k.

As an application, we present the results of the conception of a microwave coupler at the reference frequency 5 GHz. The numerical model developed remains valid to all configurations of structures that propagate the fundamental TEM mode or the quasi-TEM mode.



▲ **Figure 1. Cross section of a coupled line with inhomogeneous microstrip.**

Introduction

The analytical characterization of coupling between two coupled lines with losses and inhomogeneous microstrip is a difficult task. Numerical methods are appropriate to solve this problem. To reach this objective numerically, it is necessary to evaluate the characteristic impedances of the even and odd modes and then the primary parameters L, C, R and G of the equivalent electronic circuit of the structure.

We are interested in the numerical characterization of the coupling coefficient k and the primary parameters of two coupled lines with inhomogeneous microstrips by using the finite element method. Using this technique, we determine the influence of the electrical and the geometric parameters of the analyzed structure. Finally, we present the design of a microwave coupler.

Coupled microstrip lines

A microwave coupler has been analyzed and tested with a coupled line composed of two inhomogeneous microstrips [1]. The electrical properties of the coupler with a TEM mode or a quasi-TEM mode coupled line can be described in terms of even (Z_{0e}) and odd (Z_{0o}) mode impedances. Various numerical techniques can be used to determine the accurate characteristic impedances of the TEM and quasi-TEM coupled lines [2, 3].

In this article, we present a numerical model that analyzes a more complex coupled line. For example, we analyze Figure 1, a structure formed by two inhomogeneous microstrip line without shielding.

Figure 1 shows the cross section of a coupled line with an inhomogeneous microstrip shielded with two parallel plates considered as a perfect electric conductor (PEC). Each line consists of a conductor with the width w and the depth t. A dielectric material with permittivity ε_r (μ_r=1) fills the inside of the lines. The two lines are separated by the distance s_1.

Numerical resolution

The study in the electrostatic domain of the structures shown in Figure 1 is based on the solution of Laplace's equation in two dimensions for the two modes (even and odd):

$$div\left[\varepsilon_r \nabla_t V(x,y)\right] = 0 \quad (1)$$

Even mode:

V = 1 V on the two conductors.
V = 0 on the shield.

Odd mode:

V = 1 V on one conductor.
V = –1 V on the other conductor.
V = 0 on the shield.

The solution for this equation is found by using the finite element method (FEM) [4]. The solution for Figure 1 represents the distribution of the potential V at the different mesh nodes of the structure. When the potential V is known, we can calculate the even and odd mode impedances.

Odd and even mode characteristic impedances

The lossless lines theory allows us to determine the electrical field and the magnetic field from the potential V. The electrical energy W_{em} accumulated in the structure is calculated from the electrical field. All of the characteristic impedances (for the two modes) are deduced easily from the electrical energy W_{em}. Consequently, it is important that the exact potential V is found [4]. In the following paragraphs, we show how to study a given mode.

Electrical field — The electrical field is found through simple derivation from the potential V, using the expression

$$\vec{E}_t = -gr\vec{a}d_t(V) \quad (2)$$

The subscript t indicates the cross section of the structure.

Electrical energy — When the structure is composed of several materials of permittivity ε_{ri} (i = 1 to n), the electrical energy is deduced from the electrical field through the relationship

$$\overline{W}_{em} = \frac{1}{4}\sum_{i=1}^{n}\left(\iint \varepsilon_0 \varepsilon_{r_i} \times \vec{E}_{t_i} \times E_{ti}^{*} dxdy\right) \quad (3)$$

Capacitance per unit length — This is deduced directly from the electrical energy:

$$C = \frac{4\overline{W}_{em}}{(V_1 - V_2)^2}(F/m) \quad (4)$$

where V_1 and V_2 represent the fixed potential of the conductors.

Effective permittivity — This is calculated by obtaining the ratio of the electrical energy per unit length accumulated in the inhomogeneous structure and the energy accumulated in the same empty structure.

$$\varepsilon_{eff} = \frac{\iint \varepsilon_0 \varepsilon_r \vec{E}_{t0} \times \vec{E}_{t0}^{*} dxdy}{\iint \varepsilon_0 \vec{E}_{t1} \times \vec{E}_{t1}^{*} dxdy} \quad (5)$$

Characteristic impedance — This is calculated using

$$Zc = \frac{1}{v\varphi C}(\Omega) \quad (6)$$

where

$$v_\varphi = \frac{3\times 10^8}{\sqrt{\varepsilon_{eff}}} en(m/s)$$

Coupling coefficient — When the even and odd mode characteristic impedances are known, we calculate the coupling coefficient k, using the relationship:

$$k = \frac{Z_{0e} - Z_{0o}}{Z_{0e} + Z_{0o}} \quad (7)$$

Primary parameters

The coupling capacitance γ and the mutual inductance M are deduced from the coupling coefficient k.

$$k = \frac{\gamma}{C_0} = \frac{M}{L} \quad (8)$$

Equation 8 allows us to determine the matrices L, C, R and G using the formulas:

$$[L] = \begin{bmatrix} L & M \\ M & L \end{bmatrix}, \ [C] = \begin{bmatrix} C_o + \gamma & -\gamma \\ -\gamma & C_o + \gamma \end{bmatrix} \quad (9a)$$

and

$$[R]=\begin{bmatrix} R & O \\ O & R \end{bmatrix}, \quad [G]=\begin{bmatrix} G & O \\ O & G \end{bmatrix} \tag{9b}$$

L, C_o, R and G are the isolated line parameters calculated numerically using the finite element method.

FEM results

Using the presented theory, we established CAO programs to calculate the coupling coefficient and the matrices $[L]$, $[C]$, $[R]$ and $[G]$ of the coupled lines. All parameters depend on the features and properties of the dielectrics in which the line conductors are embedded.

When the matrices $[L]$, $[C]$, $[R]$ and $[G]$ of the structure are determined, we analyze each structure for a coupler using an adapted numerical model.

Inhomogeneous and coupled microstrip lines between two parallel plates

In order to validate our numerical results, we first studied the inhomogeneous structure presented in Figure 1, already analyzed by the moment method (MM) [1]. The features of this structure are:

- Strip width — $w = 0.85$ mm
- Strip thickness — $t = 0.0254$ mm
- Cover height — $b = 5$ mm
- Substrate thickness — $h = 0.85$ mm
- Distance — $s = 2$ mm
- Separation width — $s_1 = 0.25$ mm
- Substrate permittivity — $\varepsilon_r = 9$
- Substrate loss tangent — $t_{gd} = 0.0001$
- Reference frequency — $f = 5$ GHz
- Conductivity — $s = 5.65 \times 10^7 (\Omega m)^{-1}$

The equipotential lines of the even and odd mode are illustrated in Figure 2.

The primary parameter values obtained are:

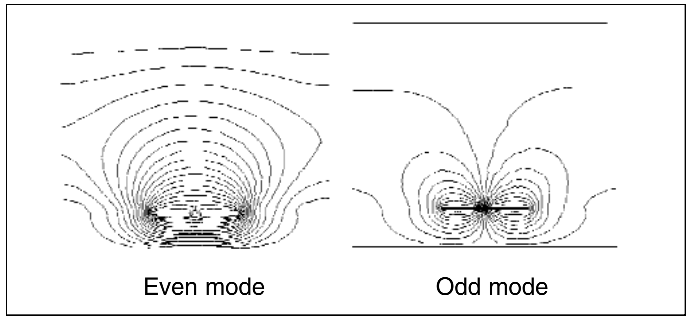
$Z_{0e} = 64.4893\ \Omega$
$Z_{0o} = 34.9464\ \Omega$
$k = 0.297105$

$$[L]=\begin{bmatrix} 419.521 & 124.642 \\ 124.642 & 419.521 \end{bmatrix}(nH/m)$$

$$[C]=\begin{bmatrix} 205.916 & -47.1655 \\ -47.1655 & 205.916 \end{bmatrix}(pF/m)$$

$$[R]=\begin{bmatrix} 19.54 & 0 \\ 0 & 19.54 \end{bmatrix}(\Omega/m)$$

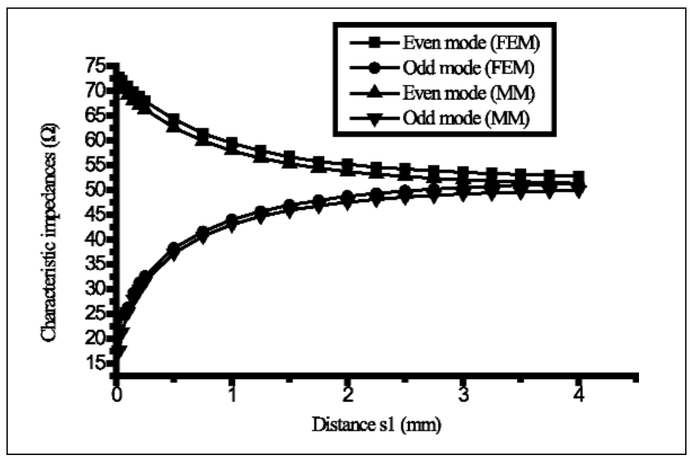
$$[G]=\begin{bmatrix} 498.729 & 0 \\ 0 & 498.729 \end{bmatrix}(\mu S/m)$$



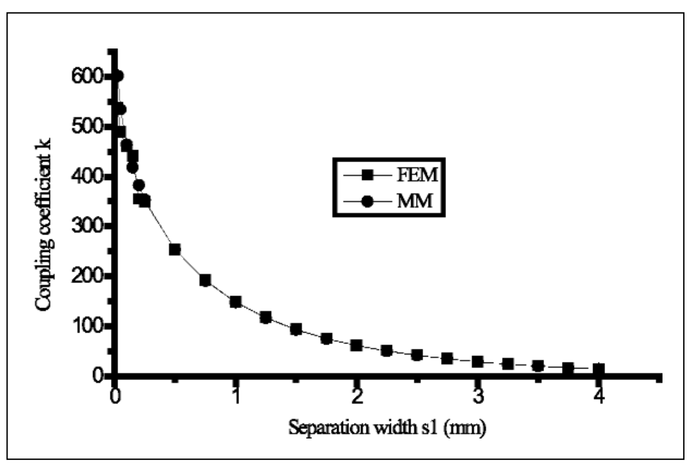
▲ **Figure 2. Equipotential lines of the even and odd modes.**

For the same geometrical and electrical parameters, we obtained the following results using the moment method (MM) [1]:

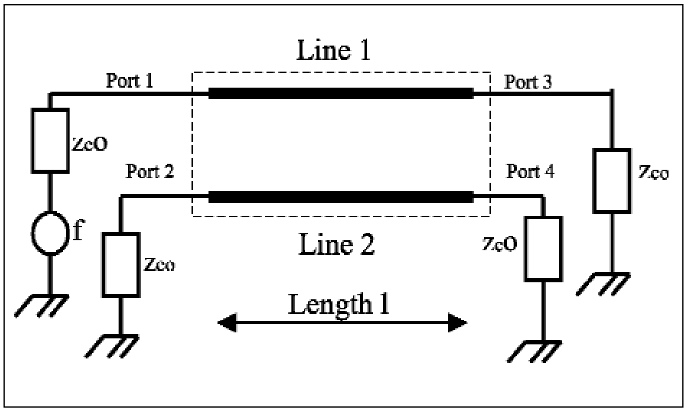
$Z_{0e} = 63.86\ \Omega$
$Z_{0o} = 34.71\ \Omega$
$k = 0.295729$



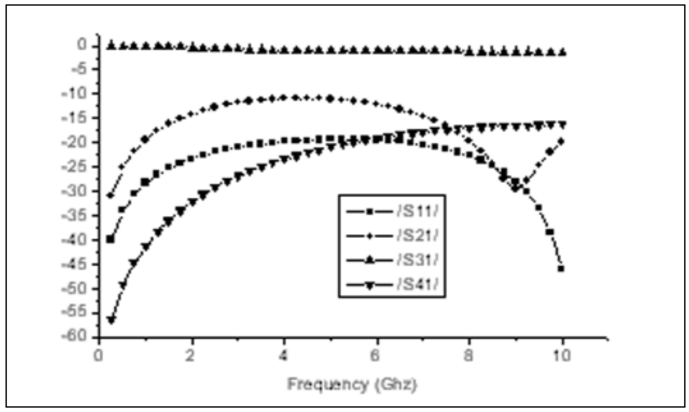
▲ **Figure 3. Impact of the separation width on the even and odd mode characteristic impedances.**



▲ **Figure 4. Impact of the separation width on the coupling coefficient.**



▲ **Figure 5. Structure of a four-port coupler.**



▲ **Figure 6. Scattering parameters of the coupler.**

$$[L]=\begin{bmatrix}390.1 & 138.6\\ 138.6 & 390.1\end{bmatrix}(nH/m)$$

$$[C]=\begin{bmatrix}170.1 & -40.03\\ -40.03 & 170.1\end{bmatrix}(pF/m)$$

$$[R]=\begin{bmatrix}23.13 & 0\\ 0 & 23.13\end{bmatrix}(\Omega/m)$$

$$[G]=\begin{bmatrix}585.9 & 0\\ 0 & 585.9\end{bmatrix}(\mu S/m)$$

Comparing the results, we find that our results correlate with those already published.

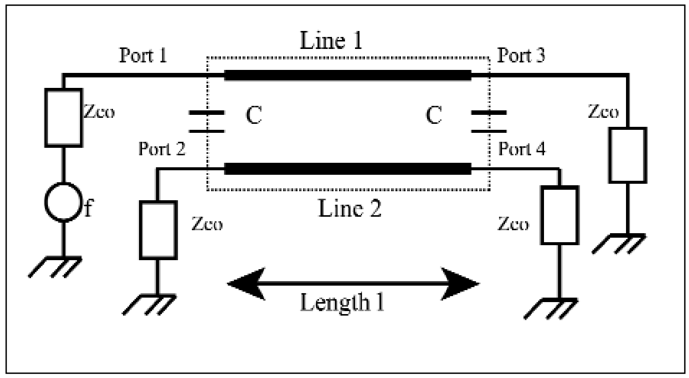
Figure 3 shows the dependence of the separation width on the even and odd mode characteristic impedances. Figure 4 illustrates the influence of the separation width on the coupling coefficient. It demonstrates an excellent agreement between our results and those obtained by the moment method.

Design of a directional coupler at 5 GHz

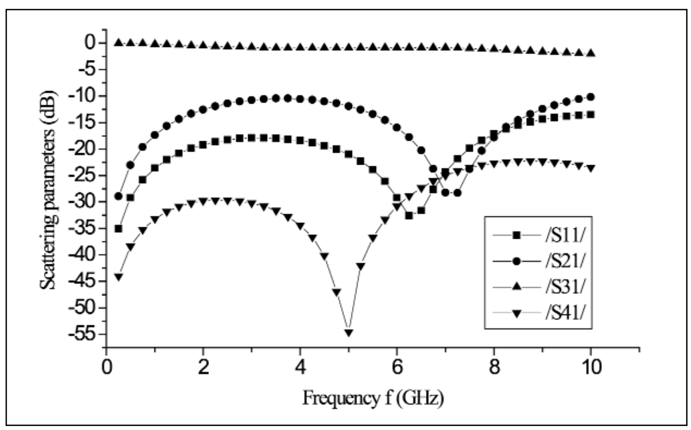
Figure 5 presents the structure of a four-port coupler. All the ports of the coupler are matched with Z_{co} = 50 Ω. For a length l = 6.4 mm, the resulting scattering parameters (with respect to 50 Ω) are plotted in Figure 6 in the band frequency (250 MHz, 10 GHz).

At 5 GHz, the coupling (–20 log IS12I) is 11.01 dB and the isolation (–20 log IS14I) is 20.35 dB.

In order to modify the response of this coupler, we connect the two signal conductors at each end of the coupler with discrete capacitors (C = 0.1 pF), as shown in Figure 7. The response of this coupler is shown in Figure 8. Much better isolation is seen than in the previous case (Figure 6), where at the reference frequency 5 GHz the isolation is 54.58 dB.



▲ **Figure 7. Coupler with compensating capacitances.**

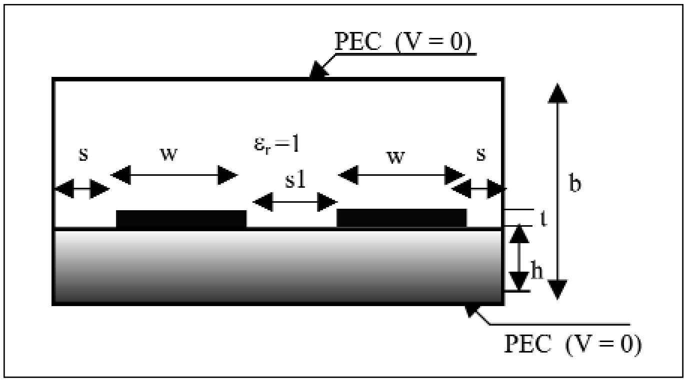


▲ **Figure 8. Influence of the frequency on the response of the coupler.**

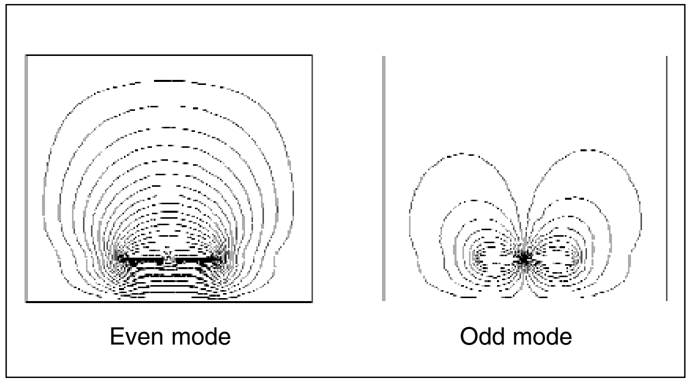
Shielded and coupled microstrip lines

We analyzed the coupled line (Figure 9), composed of two inhomogeneous and shielded microstrips, with the same geometrical and electrical parameters as in the last case. The analysis of this structure with the finite element method gives the following results (the primary parameters values obtained are displayed below):

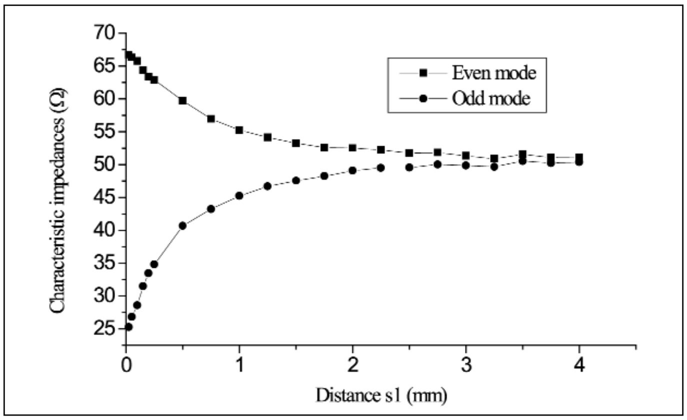
Z_{0e} = 62.8792 Ω
Z_{0o} = 34.8038 Ω
k = 0.287414



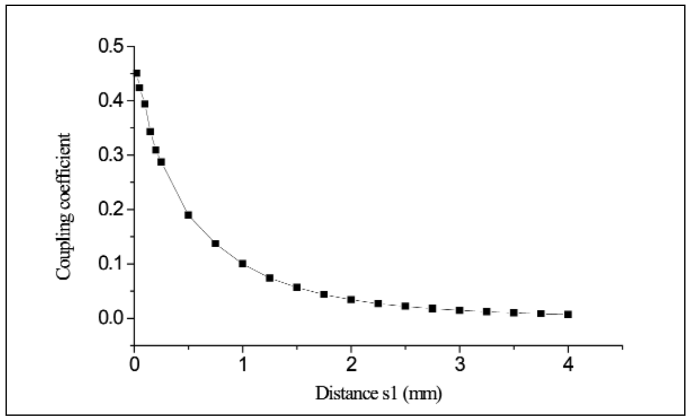
▲ **Figure 9. Cross section of the coupled line.**



▲ **Figure 10. Equipotential lines of the even and odd modes.**



▲ **Figure 11. Impact of the separation width on the even and odd mode characteristic impedances.**



▲ **Figure 12. Impact of the separation width on the coupling coefficient.**

$$[L]=\begin{bmatrix}397.016 & 114.108\\ 114.108 & 397.016\end{bmatrix}(nH/m)$$

$$[C]=\begin{bmatrix}205.518 & -45.8818\\ -45.8818 & 205.518\end{bmatrix}(pF/m)$$

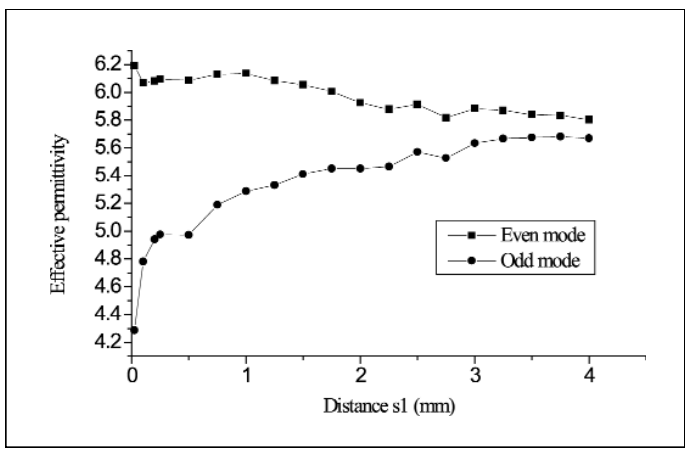
$$[R]=\begin{bmatrix}18.209 & 0\\ 0 & 18.209\end{bmatrix}(\Omega/m)$$

$$[G]=\begin{bmatrix}501.513 & 0\\ 0 & 501.513\end{bmatrix}(\mu S/m)$$

The dependence of the separation width on even and odd mode characteristic impedances is shown in Figure 11. Figure 12 illustrates the impact of the separation width on the coupling coefficient. The dependence of the separation width on the effective permittivity of the structure is shown in Figure 13.

Finally, we analyzed the same directional coupler at the reference frequency 5 GHz, this time using two shielded and inhomogeneous microstrip lines.

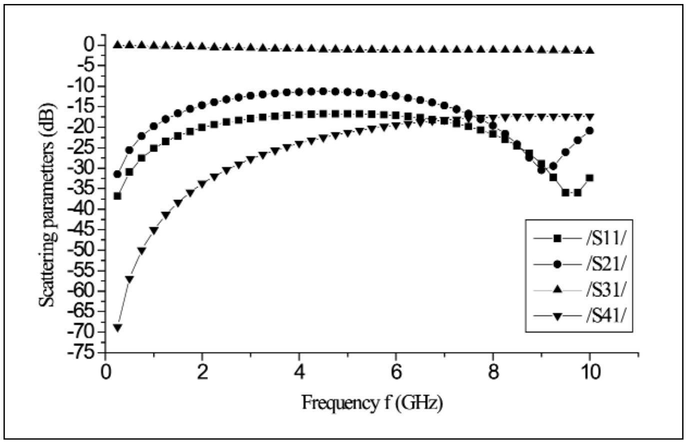
For a length l = 6.4 mm, the resulting scattering parameters (with respect to 50 Ω) are plotted in Figure 14 in the band frequency (250 MHz, 10 GHz).



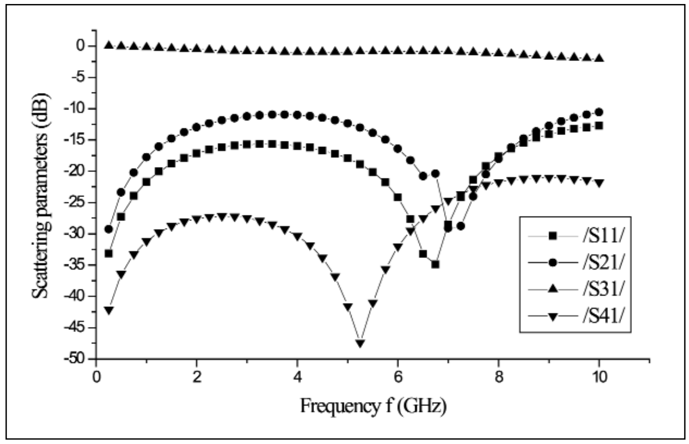
▲ **Figure 13. Influence of the separation width on the substrate permittivity.**

At 5 GHz, the coupling (–20 log |S12|) is 11.44 dB and the isolation (–20 log |S14|) is 21.31 dB.

Figure 15 shows the response of this coupler when a discrete capacitor (C = 0.1 pF) is connected at each end of the coupler. In this case the isolation improves to 41.57 dB.



▲ **Figure 14. Influence of the frequency on the response of the coupler.**



▲ **Figure 15. Influence of the frequency on the response of the coupler.**

Conclusion

This article discussed the design of a microwave coupler using lines with inhomogeneous microstrips. In order to achieve this design, it was necessary to determine the electromagnetic parameters of the system. In the band of frequency (250 MHz to 10 GHz), the problem is approximated by the resolution of Laplace's equation. Its solution was made by the finite element method, which allows the determination of the electromagnetic parameters (for example, coupling coefficient, characteristic impedances) of the structure composed of two inhomogeneous microstrip lines. The result of the CAO program shown here correlates to those that have previously been published.

The numerical model developed is valid for analyzing all configurations of multiline structures propagating the fundamental TEM or quasi-TEM modes.

All the curves presented in this paper, taking into account the influence of the electrical and geometrical parameters, prove the interest of the CAO program developed. The association of analytical functions to these curves remains possible. ■

References

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