Generalized Truncated Time Functions for SAW Filter Synthesis

This signal processing window function technique can improve stopband performance by 6 dB per octave or more

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Truncated time functions with good sidelobe behavior in frequency domain are often used in the synthesis of SAW filters. A wide range of design requirements for sidelobe levels and shape factors demand an exhaustive range of efficient truncated time functions to suit each specification. There are several time functions described in the literature which are used for SAW filter synthesis. Among them, the cosine series functions offer stopband rejection and mainlobe width which increase with the increase in the order of the series.

A general set of new truncated series is described in this paper, which facilitates derivation of an innumerable number of truncated time functions to obtain a range of sidelobe levels and mainlobe widths in addition to those obtained by the cosine series. As an example, a set of truncated time function series is derived from the generalized function series and frequency response plots of the series up to the third order are given. The sidelobe levels and mainlobe widths of the series are different from those due to cosine series, adding further flexibility in designing SAW filters.

Introduction

SAW filter synthesis procedures with non-iterative techniques use appropriate truncated time functions to obtain desired frequency response. In general, as the order of the function increases sidelobe levels decrease and mainlobe width increases [1-4] for a given impulse duration. A plot of time-bandwidth product versus sidelobe level is often used to compare various window functions [1]. The asymtotic decay is another parameter used for comparison of these functions for their far-off frequency response [2]. Both of the above criteria are used to characterize the window functions described in this paper.

Derivation of the generalized set of series

The generalized set of truncated series is defined as follows.

$$g_{Na}(t) = q(t)\sum_{n=0}^{N-1} a_n \cos\left(\frac{2n\pi t}{T}\right); \quad \text{for } |t| \le \frac{T}{2}$$
$$= 0; \quad \text{for } |t| > \frac{T}{2} \tag{1}$$

$$g_{Nb}(t) = q(t)\sum_{n=0}^{N-1} b_n \cos\left(\frac{(2n+1)\pi t}{T}\right); \text{ for } |t| \le \frac{T}{2}$$
$$= 0; \quad \text{for } |t| > \frac{T}{2} \tag{2}$$

where the coefficients a_n and b_n are positive and real numbers to be determined and $g_{Na}(t)$and $g_{Nb}(t)$ are the truncated time functions which exist only for the duration from $-T/2$ to $+T/2$ and contain N terms in the series. The function $q(t)$ is continuous and even function in the span $-T/2$ to $+T/2$ and has a Fourier transform $Q(f)$. Thus $q(t)$ and $Q(f)$ are related by Fourier transform pair as given below.

$$Q(f) = \int_{-\infty}^{\infty} q(t)e^{-j2\pi ft}dt \tag{3}$$

$$q(t) = \int_{-\infty}^{\infty} Q(f)e^{j2\pi ft}df \tag{4}$$

The corresponding frequency domain functions of (1) and (2) are obtained by performing Fourier transform on (1) and (2) and are expressed as:

$$G_{Na}(f) = \sum_{n=0}^{N-1}\left(\frac{a_n}{2}\right)\left[Q\left(f - \frac{n}{T}\right) + Q\left(f + \frac{n}{T}\right)\right] \tag{5}$$

$$G_{Nb}(f) = \sum_{n=0}^{N-1}\left(\frac{b_n}{2}\right)\left[Q\left(f - \frac{m}{T}\right) + Q\left(f + \frac{m}{T}\right)\right] \tag{6}$$

where $m = (2n+1)/2$.

The functions $G_{Na}(f)$ and $G_{Nb}(f)$ can become either even or odd functions as per the definition of even and odd functions given in [1], depending on the nature of the function $q(t)$. For example if the function $q(t)$ is unity in the interval $-T/2$ to $+T/2$, then $G_{Na}(f)$ will be an odd function as it contains odd number of kernels (lobes) and $G_{Nb}(f)$ will be an even function as it contains even number of kernels and they represent the cosine series functions described by Malocha and Bishop [1]. However, if $q(t)$ is chosen as parabolic (as described in the following sections), then $G_{Na}(f)$ will become an even function and $G_{Nb}(f)$ will become an odd function.

Determination of coefficients

The coefficients a_n and b_n are determined uniquely by solving N–1 simultaneous equations which are formed by applying following criteria.

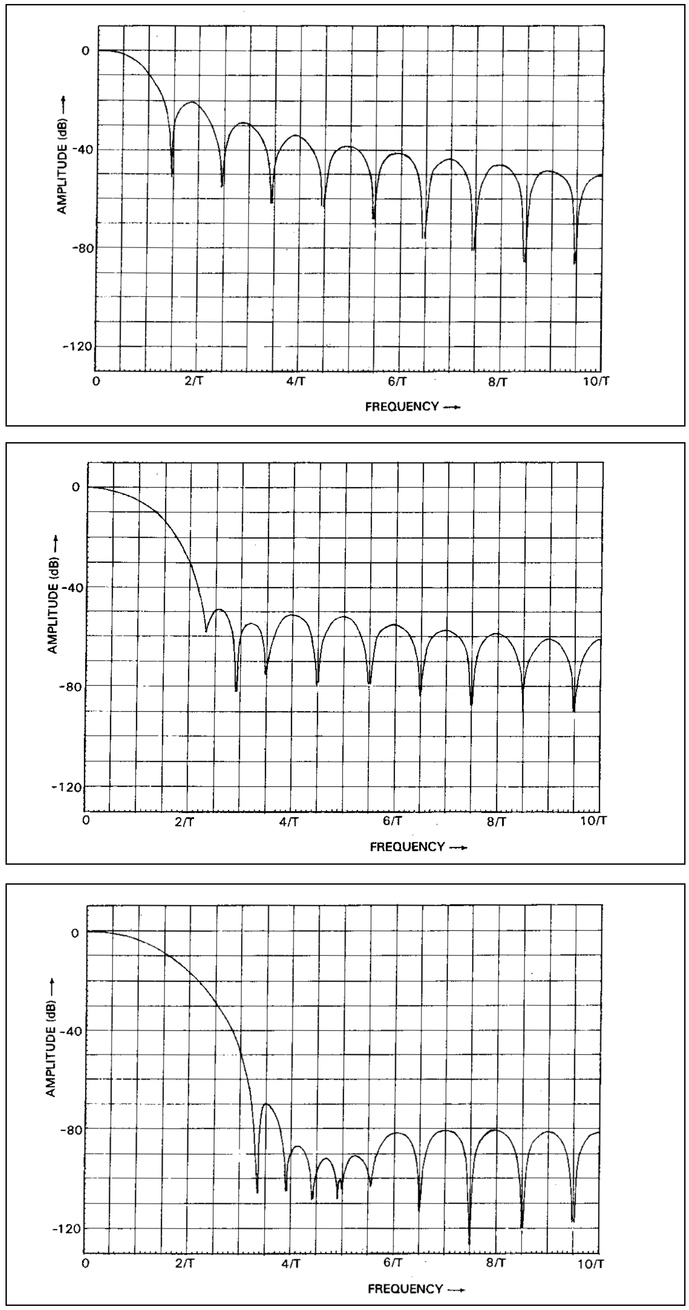
1. The coefficients a_0 and b_0 are assigned unity value for determination of unique coefficients [3].
2. The frequency functions are nulled at the first adjacent N–1 sidelobe peaks of the Nth term in the series [1, 2, 3].

The sidelobe peaks of Nth term in the series are chosen, since the Nth term contributes maximum amplitude to the sidelobes compared to the remaining N–1 terms of the series [3]. The Nth terms in the frequency functions $G_{Na}(f)$ and $G_{Nb}(f)$ are denoted by $S_{Na}(f)$ and $S_{Nb}(f)$ respectively and are expressed as:

$$S_{Na}(f) = Q[f - \{(N-1)/T\}] + Q[f + \{(N-1)/T\}] \tag{7}$$

$$S_{Nb}(f) = Q[f - \{(2N-1)/2T\}] + Q[f + \{(2N-1)/2T\}] \tag{8}$$

The first N–1 adjacent sidelobe peaks of (7) and (8) are found by differentiating these functions with respect to frequency and equating to zero. At these points the frequency function $G_{Na}(f)$ (or $G_{Nb}(f)$ as the case may be) is forced to zero, giving N–1 simultaneous equations which are to be solved to obtain the coefficients a_1 to a_N (or b_1 to b_N in the case of $G_{Nb}(f)$).



▲ **Figure 1. Frequency response of the window function G_{Na} for $N = 1$ (top), $N = 2$ (center) and $N = 3$ (bottom).**

An example function

As an example consider the function $q(t)$ represented by a parabola as follows.

$$q(t) = 1-(4t^2/T^2);\ \text{for } |t| \le T/2$$
$$= 0;\ \text{for } |t| > T/2 \tag{9}$$

The Fourier transform of $q(t)$ is:

$$Q(f) = 2T\{\sin(\pi f T) - [\pi f T \cos(\pi f T)]\}/(\pi f T)^3 \tag{10}$$

At $f = 0$, the function $Q(f)$ is indeterminate and the limiting value of $Q(f)$ as f tends to zero is found to be $2T/3$. The equations (7) and (8) are now simplified with the known expression for $Q(f)$, and the following condition results for the sidelobe peak points of $S_{Na}(f)$:

$$\tan(x) = \frac{3wz\left(w^3 + z^3\right)}{3\left(w^4 + z^4\right) - w^2z^2\left(w^2 + z^2\right)} \tag{11}$$

where $x = \pi f T$

$w = \pi f T - (N-1)\pi$

$z = \pi f T + (N-1)\pi$

Similarly, the condition for the sidelobe peaks of $S_{Nb}(f)$ is found to be :

$$\tan(x) = \frac{(u+v)\left[u^2v^2 - 3\left(u^2 + v^2\right)\right]}{3uv\left(u^2 + uv + v^2\right)} \tag{12}$$

where $x = \pi f T$

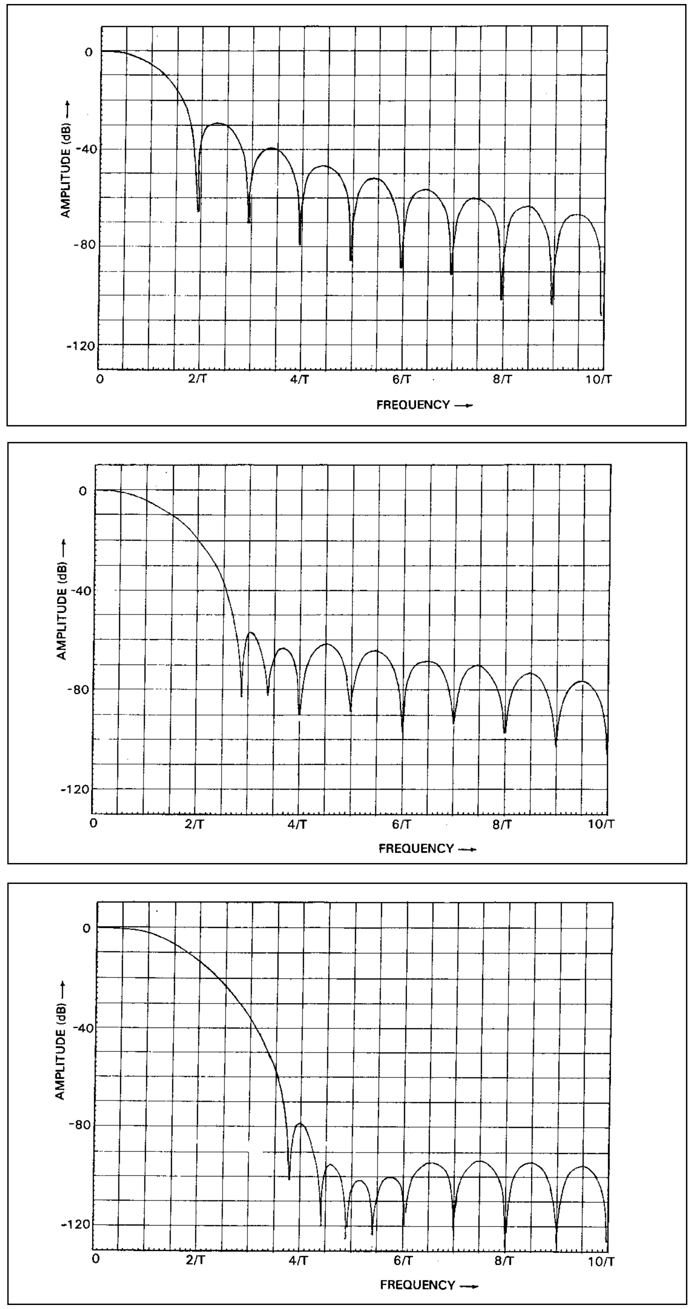
$u = \pi f T - [(2N-1)(\pi/2)]$

$v = \pi f T + [(2N-1)(\pi/2)]$

The peak frequencies of $S_{Na}(f)$ and $S_{Nb}(f)$ are found out by numerically solving the equations (11) and (12) for N up to 3. These peak points are given in Table 1. Simultaneous equations are formed by forcing the frequency functions to zero at these $N-1$ peak points. The coefficients a_1 to a_N (and b_1 to b_N) are determined by solving these simultaneous equations. The coefficients for N up to 3 for both the series are given in the Table 1. The plots of frequency functions G_{Na} and G_{Nb} for N up to 3 are shown in Figures 1 and 2. Table 2 lists the mainlobe widths and the sidelobe levels obtained for the above functions. Comparing the frequency plots of G_{Na} and G_{Nb} with the corresponding plots of cosine series in reference [1], we see that the functions described in this paper have better asymtotic decay of sidelobes than the functions given in [1]. By carrying out analysis of asymtotic decay as described in [5], we see that the cosine series functions $HN1$ and $HN2$ presented in [1, 3] have 6 dB/octave and 12 dB/octave decay of sidelobes, where as the functions, G_{Na} and G_{Nb}, described in this paper have 12 dB/octave and 18 dB/octave decay of sidelobes.

Conclusions

A generalized set of truncated time functions useful for SAW filter synthesis is defined which generates innumerable number of truncated sets by just varying $q(t)$. An example set is derived by making $q(t)$ parabolic and the coefficients of the series are determined. Plots of the example set are shown for N up to 3. The frequency responses of the functions are analyzed in terms of mainlobe width, sidelobe level and asymtotic decay.



▲ **Figure 2. Frequency response of the window function G_{Nb} for N = 1 (top), N = 2 (center) and N = 3 (bottom).**

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	N	Location of peaks 1	2	Coefficients for: $n=1$	$n=2$	$n=3$
G_{Na}	1	—	—	1	—	—
	2	$2.851/T$	—	1	0.67084	—
	3	$3.845/T$	$4.904/T$	1	1.09081	0.1415
G_{Nb}	1	—	—	1	—	—
	2	$3.315/T$	—	1	0.21495	—
	3	$4.325/T$	$5.384/T$	1	0.37493	0.03758

▲ **Table 1. Peak frequency points and coefficients of functions G_{Na} and G_{Nb}.**

	N	Mainlobe width	Sidelobe level	Decay (dB/octave)
G_{Na}	1	$\pm 1.4/T$	–21 dB	–12
	2	$\pm 2.3/T$	–49 dB	–12
	3	$\pm 3.3/T$	–71 dB	–12
G_{Nb}	1	$\pm 1.9/T$	–29 dB	–18
	2	$\pm 2.8/T$	–57 dB	–18
	3	$\pm 3.8/T$	–80 dB	–18

▲ **Table 2. Mainlobe widths and sidelobe levels for the functions G_{Na} and G_{Nb}.**

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