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Behaviour of Reflected Pulses along Cables

Sine waves and pulses have similar propagation and reflection behaviour along cables. The phenomenon of total reflection and of matching of pulses is perhaps better understood by using the mechanical analog of perturbations along a length of rope. Also, the calculation of reflection coefficient with ohmic loads, is easier to explain with pulses than with sine-wave energy. The electrical process at relatively low frequencies and long propagation times is demonstrated more simply with a sufficiently long cable.

1. TOTAL REFLECTION AND MATCHING USING A MECHANICAL MODEL

A mechanical impulse is caused on a length of rope by the means shown in **fig. 1**, where the hammer may be regarded as a pulse generator. The pulse travels, from left to rigth, down the line until its end. What happens then is determined by three cases:

a) The rope is tied to an immovable object (fig. 1 a):

The pulse will be reflected at the rope's end with the same amplitude but at the opposite polarity to that of the incident wave. The end is anchored to an immovable object; therefore, no energy can be imparted and the incident energy is reflected in its entirety. The polarity change is brought about because the incident and reflected waves simply cannot exist together at the same time because the rope's end is tied to an immovable object.

b) The end of the rope is free (fig. 1 b):

The end of the rope is free inasmuch that it is held in position by a very much thinner length of cotton, which allows the rope's end to move when subjected to a mechanical stimulus. The perturbation travels down the rope, and again, is reflected in its entirety but as the end of the rope is free to assume any position, the reflected wave has the same polarity as the incident wave. The wave is totally reflected as in the first case but if a high-speed photograph was taken at the moment the incident pulse reached the end of the rope, it would show that the end flies up to a position which is twice that of the amplitude of the incident pulse. This indicates that at this instant the total

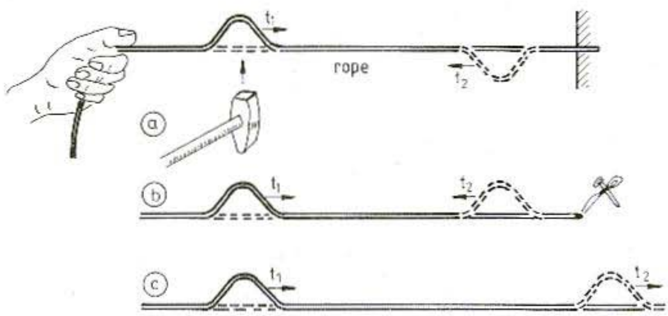

Fig. 1:
Propagation of pulses along a rope,
a) fixed end,
b) loose end
c) Rope infinately long

amplitude consists of the sum of both the incident and the reflected pulses as they overlap.

c) The rope is fastened in a medium (fig. 1 c): The medium is considered to be so pliable that all the energy in the pulse is dissipated as heat. In this case, no energy can be reflected at all. The same effect would occur if the rope were infinately long. The impulse would travel on and on until it lost all its energy in frictional heat. Reflected waves cannot occur. In the electrical analogy it is called a "matched" condition.

2. TOTAL REFLECTION AND MATCHING USING CABLES

The mechanical analogy will now be dispensed with by using a cable instead of a rope and an electrical square wave generator instead of a hammer. This may be seen in the series represented in **fig. 2**. The **"fixed end"** here is shown in

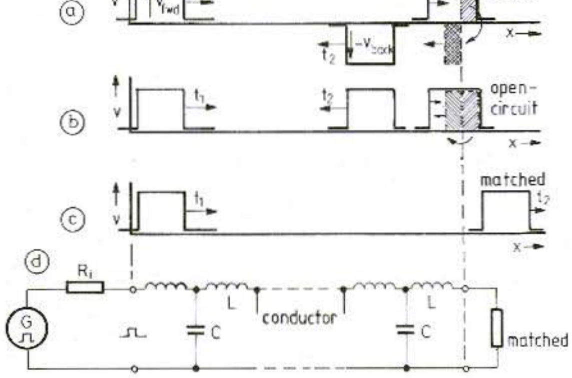

Fig. 2:
Propagation of pulses along a lossless conductor
a) short-circuited
b) open-circuited
c) matched
d) equivalent circuit of a conductor

fig. 2 a and is represented by a short-circuited cable-end. At a short-circuit, no voltage exists across it (voltage node). This has the effect that the incident pulse develops a voltage which is of the same amplitude but of opposite phase. Since it is at the end of the line, the pulse has no option but to travel, with reversed phase, back down the line from whence it came. The returning wave transports, a practically reactive power, back into the generator. If the generator is not matched to the cable, the pulse will be reflected again, this time by the generator's internal impedance.

Fig. 2 b shows the case of the **"loose"** or **"open-ended"** line. Ignoring the possibility of radiation from the cable end, it may be seen that no energy can be dissipated in an open-circuit. At the open end of the cable the pulse can develop until it has the same amplitude and phase as the incident pulse. At the moment of encountering the open-circuit the instantaneous voltages rise to double that of the incident pulse alone. This can easily be seen with the aid of an oscilloscope. This doubled voltage corresponds to that of the generator when open-circuited. Along the line, each individual input has an amplitude of half that of the generator open-circuit voltage (assuming that the generator output resistance R_i is the same as the characteristic impedance Z_0 of the cable).

The **"matched case"** is shown in **fig. 2 c**, which means in this, the electrical analogy, that at the end of the line the pulse finds the same conditions of voltage and current as was encountered along the whole length of the line. The relationship between voltage and current on the line is determined by its inductance and capacitance and is of the form $\sqrt{L/C}$. This is also an expression for the characteristic impedance of the cable Z_0 and if it is equal in value to the load resistor $Z_0 = \sqrt{L/C} = R_L$ then all the power in the pulse will be dissipated in this load termination at the end of the cable. This is analogous to the mechanical case of the infinately long rope. The electrical energy of the travelling pulse would eventually disappear in heating the small, but ubiquitous, copper and dielectric loss resistances along the line.

In **Figs. 2 a** and **2 b** the hatched areas at the cable end show how the reflected impulse development may be imagined. The incident pulse just cannot grow out of the end of the cable. The projecting piece x can be regarded as folding back upon itself in the open-circuited case and also in the short-circuited case but this time with an inverted polarity. The superimposition of incident and return impulses, is however, not shown in fig. 2. This voltage doubling be observed on an oscilloscope by monitoring a point along the line where a reflected pulse meets the next incident pulse in the pulse train sequence.

2.1. Partial Reflection

It is plausible that between the extremes of short-circuit and open-circuit there will be a case of partial reflection. If at a point along the line there is a discontinuity causing a reflection, the point is known as a "line fault". Only a fraction will continue down the line and another fraction is returned to the generator. The same sort of thing occurs when the line is terminated by a load resistance. The load resistance dissipates part of the energy as heat and reflects the rest back towards the generator, the amplitude being smaller than that of total reflection. This condition is known as a "mismatch". The mismatch can tend towards being a short-circuit or it can tend towards being an open-circuit according to whether the termination is less than Z_0 or greater than Z_0 respectively. The relationship between the amplitude of the return pulse to that of the incident pulse is known as the "reflection coefficient" (**fig. 3**).

2.2. Example

A pulse generator, internal impedance $R_i = 50\ \Omega$, has an unterminated output voltage $\hat{V}_0 = 20$ V. A resistive load of 75 Ω is connected to it via a length of "lossless" cable having a characteristic impedance of 50 Ω. What is the magnitude of the incident voltage $\hat{V}_{FWD}$ and of the return voltage $\hat{V}_{BACK}$?

Solution:

Since the generator, at first, does not "know" yet the terminating resistor at the end of the cable, the

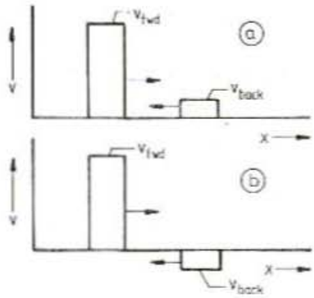

Fig. 3: Partial reflection
a) tending to open-circuit
b) tending to short-circuit
Magnitude of reflection factor
$r = \hat{V}_{BACK} / \hat{V}_{FWD}$

incident pulse amplitude is determined by the proportional impedance existing between the generator and the cable. The generator "sees" the cable impedance (but not the load impedance).

Since $R = Z_o$ then the voltage splits equally across these impedances (i. e. $\hat{V}_{FWD} = \hat{V}_o / 2 = 10\ V$). A current pulse $\hat{I}_{FWD}$ is also associated with the incident wave which has an amplitude $\hat{V}_{FWD} / Z_o = 10\ V / 50\ \Omega = 0.2\ A$. Owing to the mismatch at the cable-end, the forward pulse is partially reflected and a voltage $\hat{V}_{BACK}$ is formed together with a current $\hat{I}_{BACK} = \hat{V}_{BACK} / Z_o$. What their magnitude and sign is, depends upon the mismatch. For this particular case, tending towards open-circuit termination ($R_L > Z_o$), the forward and return pulses have the same polarity voltage and they add $\hat{V}_{FWD} + \hat{V}_{BACK}$ across the load resistor. The forward and return currents, however, have opposite polarities (high R_L therefore small load current) and the total composite current through the load is:

$$I_L = \hat{V}_{BACK} (\hat{V}_{FWD} / Z_o - \hat{V}_{BACK} / Z_o)$$

The resultant $\hat{I}_{BACK}$ forms so that ohm's law is fulfilled at the load.

$$\hat{I}_{BACK} = \hat{V}_{FWD} + \hat{V}_{BACK} = (\hat{V}_{FWD} / Z_o - \hat{V}_{BACK} / Z_o)\ R_L$$

I. e. the total voltage at the cable end = the total current times the load resistance.

After putting in figures, a simple equation remains, which will give the unknown, namely $\hat{V}_{BACK}$

$$(10\ V + \hat{V}_{BACK}) = (10\ V / 50\ \Omega - \hat{V}_{BACK} / 50\ \Omega) : 75\ \Omega$$

re-arranging:

$$\hat{V}_{BACK} = 10\ V\ (75\Omega - 50\Omega) / (75\ \Omega + 50\ \Omega) = 2\ V$$

2.3. Reflection Coefficient

The fraction in the above example gives the factor which, when multiplied by the incident voltage, results in the amplitude of the return voltage. This factor is known as the reflection coefficient r. In general, the formula in terms of an ohmic load:

Reflection coefficient $r = (R_L - Z_o) / (R_L + Z_o)$.

Referring to fig. 2 c $R_L = Z_o$ and therefore the reflection coefficient $r = 0$. (no reflection). In fig. 2 b, $R_L = \infty$ therefore $r = 1$ (total reflection). The situation depicted in fig. 2 a results also in total reflection but $R_L = 0$, therefore $r = -1$. The sign change indicates that although forward and return voltages posses equal amplitudes, i. e. total reflection, the polarity of the return pulse is negative compared with that of the incident pulse. (see also fig. 1 a).

Fig. 3 shows the case of partial reflection, **fig. 3 a** shows that tending to infinate resistance, and **fig. 3 b** shows that tending to zero resistance.

Example:
In fig. 3, $\hat{V}_{FWD} = 2.5\ V$. The 50 Ω cable is terminated with 75 Ω. What is the amplitude of the reflected pulse?

Solution:

$$r = (75\ \Omega - 50\ \Omega) / (75\ \Omega + 50\ \Omega) = 0.2.$$

therefore,

$$\hat{V}_{BACK} = 2.5\ V \times 0.2 = 0.5\ V.$$

Note:
When the line losses are finite, quite large errors can be introduced by the cable attenuation. The return pulse arriving back at the generator has been attenuated twice, once on the incident journey and then on the return journey.

Example:
The complete cable has an attenuation of 1.5 dB. The forward has an amplitude of 2.5 V. The cable

has a characteristic impedance of 50 Ω and is terminated with a load resistor of 75 Ω. With what amplitude does the reflected pulse arrive back at the generator?

Solution:

The incident pulse appears at the cable end attenuated by

$$1.5 \text{ dB i. e.} \triangleq 1.19,$$
$$\hat{V}_{FWD\,(term.)} = 2.5\text{ V} / 1.19 = 2.1\text{ V}$$

as a consequence of the mismatch

$$r = (75\ \Omega - 50\ \Omega) / (75\ \Omega + 50\ \Omega) = 0.2$$

and the magnitude of the return voltage is:

$$\hat{V}_{BACK\,(term.)} = 2.1\text{ V} \times 0.2 = 0.42\text{ V}$$

Upon the return journey this voltage experiences a further attenuation of 1.5 dB

$$\hat{V}_{BACK\,(gen.)} = 0.42\text{ V} / 1.19 = 0.35\text{ V}$$

This means that, because of the attenuation the reflection coefficient $r = 0.35\text{ V} / 2.5\text{ V} = 0.14$ i. e. 14 % instead of 20 % neglecting attenuation.

3. MEASURING WITH PULSES

From the reciprocal of $c = 300 \times 10^6$ m / s (the speed of light through space) the time per metre is obtained. When the velocity through the cable is considered, it is reduced by a factor dependent upon the physical characteristics of the cable. Most coaxial cables have a velocity factor of 0.66 which means that the signal requires some 1.5 ns longer to traverse one metre of cable than it would through one metre of space. Short cables used for pulse measurement require therefore, pulses with rise-times of only a few nano-seconds and an oscilloscope with a bandwidth of a few tens of MHz. In order to reduce the requirements upon the test equipment, it is better to use a longer cable of length say, from 50 to 100 m. By this practice, the pulses may be clearly displayed using a generator with a rise-time of 0.1 to 0.5 μs and an oscilloscope of only a few MHz bandwidth.