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Formulae and Diagrams for the Approximate Calculation of Micro-Striplines

The design and calculation of stripline circuits is still, unfortunately, a specialist's territory. This is partly understandable, as the development of the circuits is more demanding upon the technological possibilities than that for normal printed circuit practice. In spite of this, it is helpful for the understanding of micro-strip circuits, if the impedances of the conductor structure can be determined from its geometry. This will enable anyone with normal radio frequency expertise to, at least, understand the circuit function.

That is the aim of the following presentation of formulae, and in particular, for the understanding and calculation of micro-stripline antennas which are described in this edition of VHF COMMUNICATIONS.

Planar microwave circuits, which are relatively easy to fabricate using etching techniques, mostly employ unsymmetrical striplines. The exact calculation of this type of circuit is very tedious, and in most instances, only possible as an approximation. In order to present a simple method of calculating microwave conducting structures, only the most important approximation formulae from the references /1, /2, /3 have been selected and presented graphically. The diagrams have been calculated based upon the most used substrate, glass-fibre re-inforced PTFE. Types include RT/Duroid 5870 and 5880 Rogers Corp. /4/ or Di-Clad 870 and 880 Keene /5/ with dielectrics constants $\varepsilon_r = 2.32$ and 2.23, resp.

The inaccuracies, consequent upon the approximation approach, are small and well within the tolerances for the dielectric constant and the thickness of the substrate materials. I found that the difference between the calculated results and the actual measured results lay within ± 3 %. This is not normally critical for simple circuits (filters and resonators), however, a correction may have to be applied. The fabrication must also be carefully controlled, as the design of the mask, application of the photo sensitive resist, the exposure, development and etching all influence the width of the conductor tracks. These manufacturing tolerances must be known and taken into account.

1. CONDUCTOR WIDTH

Fig. 1 shows a cross-sectional view of a micro-strip conductor of width W and conductor thickness t, etched from a board of dielectric thickness h and relative dielectric constant ε_r. The conducting ground plane is continuous. The calculation

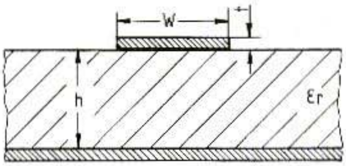

Fig. 1: Cross-section through a micro-stripline

of the conductor dimensions is accomplished by the aid of /3/ **equation (1)** using the application range $\varepsilon_r \leq 16$ and $0.5 \leq W / h \leq 20$ assuming $t = 0$ and neglecting frequency. They supply for the desired conductor characteristic impedance Z_L the ratio of track width to substrate thickness W / h

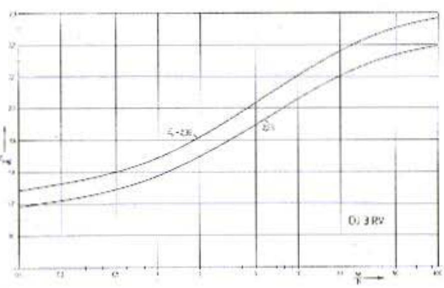

Diagram 1: Effective relative permittivity ε_{reff} as a function of W/h for an ε_r of 2.23 and 2.32

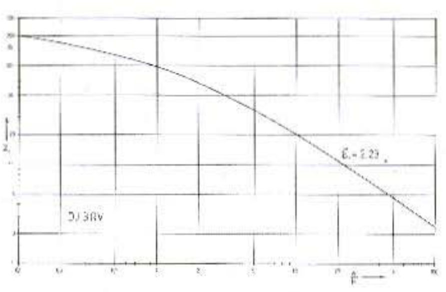

Diagram 2: Characteristic impedance Z_L of a stripline as a function of W/h for $\varepsilon_r = 2.23$. The displacement of Z_L for $\varepsilon_r = 2.32$ is small enough to be neglected and cannot therefore be shown as a seperate curve.

for the given dielectric constant ε_r of the substrate material. **Equation (2)** supplies the effective dielectric constant ε_{reff} i. e. the modified value due to lines of force fringing.

For an overall view, the values for $\varepsilon_r = 2.23$ and $\varepsilon_r = 2.32$ are depicted in **diagram 1** as a function of W / h.

The conductor characteristic impedance Z_L may be calculated with the aid of **equation (3)** from the given geometrical dimensions. These values for the impedance Z_L are plotted against the ratio W / h for $\varepsilon_r = 2.23$ in **diagram 2**. The values for the characteristic impedance Z_L at $\varepsilon_r = 2.32$ lie about 2 % lower and cannot be clearly depicted in the diagram.

The influence of the conductor track thickness t has been neglected in **equation (1) to (3)** but the error is very small for track thicknesses from 17.5 μm to 35 μm. Thick conductor tracks also very narrow tracks, exhibit greater lines-of-force fringing effects and the effective dielectric constant is therefore smaller. This factor can be taken into account with **equation (4) and (5)** from /1/. The resulting corrected value for ε_{reff}^* and conductor width W* are used in **equation (3)** to obtain an improved value for the characteristic impedance. The conditions $h >> t$; $2\,t < W$ and $t < 0.75\,(W^* - W)$ must, however, be observed.

The influence of frequency is dertermined by /2/ **equation (6)**. This supplies a frequency corrected effective dielectric permittivity $\varepsilon_{reff}(f)$ which at 10 GHz and with PTFE substrate is about 2 % higher than determined by **equation (2)**.

2. END CAPACITANCE OF AN OPEN-CIRCUIT LINE

An open-circuited micro-stripline has, at its end, a fringe field which has a capacitive effect. This tends to give the conductor a greater electrical length than its physical length by an amount d.

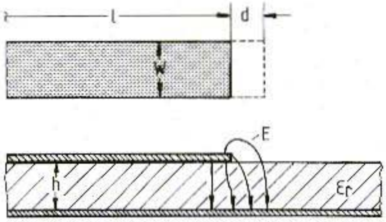

Fig. 2: Additional length d of an open-circuit stripline caused by the fringing of the E field at its end

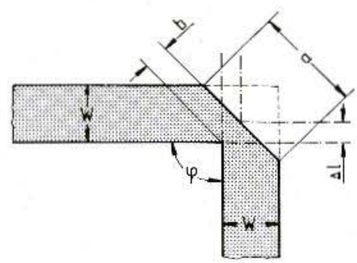

Fig. 3: Compensation of the fringing effect at a bend by cutting off the corner

Fig. 2 shows this fringing field and the resultant elongation d. This effect must be taken into account e. g. with probes or resonators in **equation (7)** from /2/ and is valid for $0.01 \leq W/h \leq 100$ and $1 \leq \varepsilon_r < 50$. The curve is shown in **diagram 3** for $\varepsilon_r = 2.23$.

3. COMPENSATING DEVICES

When designing microstrip circuits, it is frequently necessary to form an angle φ in the track to change direction. A bend is formed as shown in **fig. 3**, which also possesses a fringing field which adds an effective additional capacity. A relatively wide-band compensation method is to cut the corner as in **fig. 3**. In /1/ a corner-cut of length a = 1.8 W for angles φ = 30° to 120° is given. The width b is calculated from **equation (8a)**. My measurements showed that these approximations are effective for angles φ = 90° to 120°.

For a right-angled bend φ = 90°, the width b is determined according to /2/ also **equation (8b)**. The size of d must then be determined with **equation (7)** for an open-circuited line of width $\sqrt{2}\,W$.

The equivalent length Δl is approximately given by **equation (9)**.

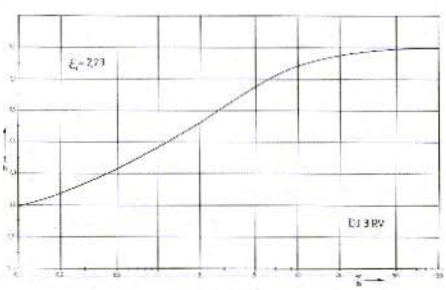

Diagram 3: Elongation of length by amount d for an open-circuited stripline for a relative permittivity of $\varepsilon_r = 2.23$

4. SYMMETRICAL BRANCHING

The reference plane displacement at a line branching section is shown in **fig. 4**. This occurs with probes, conductor dividers or hybrid couplers and is extensively covered in /1/. For a symmetrical

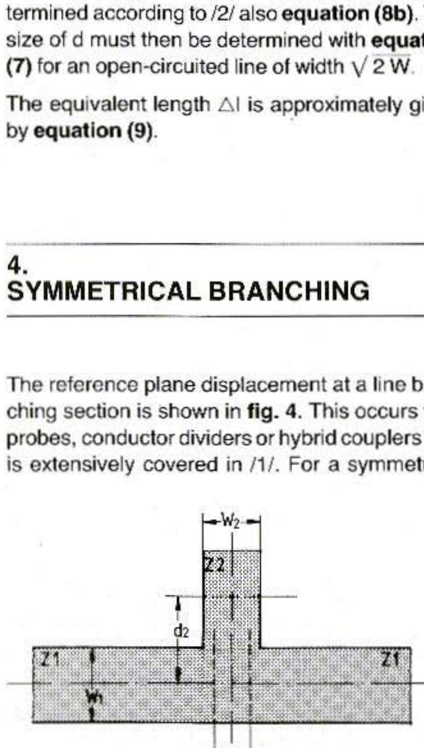

Fig. 4: Displacement of reference plane at a stripline junction

Equations:

$$\frac{W}{h} = \begin{cases} \dfrac{8}{e^{A} - 2e^{-A}} ; & \text{for } \dfrac{W}{h} \leq 2 \\[2ex] \dfrac{2}{\pi}\left[B - \ln(2B+1) + \dfrac{\varepsilon_r - 1}{2\varepsilon_r}\left(\ln B + 0{,}39 - \dfrac{0{,}6}{\varepsilon_r}\right)\right] ; & \text{for } \dfrac{W}{h} \geq 2 \end{cases} \quad (1)$$

$$\text{where } A = \frac{Z_L}{120\Omega}\sqrt{2(\varepsilon_r+1)} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0{,}23 + \frac{0{,}11}{\varepsilon_r}\right) ;$$

$$\text{and } B = \frac{60\Omega\pi^2}{Z_L \cdot \sqrt{\varepsilon_r}} - 1 ;$$

$$\varepsilon_{reff} = \begin{cases} \dfrac{\varepsilon_r + 1}{2} + \dfrac{\varepsilon_r - 1}{2}\left[\left(1 + 12\dfrac{h}{W}\right)^{-\frac{1}{2}} + 0{,}04\left(1 - \dfrac{W}{h}\right)^2\right] ; & \text{for } \dfrac{W}{h} \leq 1 \\[2ex] \dfrac{\varepsilon_r + 1}{2} + \dfrac{\varepsilon_r - 1}{2} \cdot \left(1 + 12\dfrac{h}{W}\right)^{-\frac{1}{2}} ; & \text{for } \dfrac{W}{h} \geq 1 \end{cases} \quad (2)$$

$$Z_L = \begin{cases} \dfrac{60\ \Omega}{\sqrt{\varepsilon_{reff}}} \ln\left(8\dfrac{h}{W} + 0{,}25\dfrac{W}{h}\right) ; & \text{for } \dfrac{W}{h} \leq 1 \\[2ex] \dfrac{377\ \Omega}{\sqrt{\varepsilon_{reff}}}\left[\dfrac{W}{h} + 1{,}393 + 0{,}667 \cdot \ln\left(\dfrac{W}{h} + 1{,}444\right)\right]^{-1} ; & \text{for } \dfrac{W}{h} \geq 1 \end{cases} \quad (3)$$

$$\varepsilon_{reff}^{*} = \varepsilon_{reff} - \frac{(\varepsilon_r - 1)t}{4{,}6\sqrt{W \cdot h}} \quad (4)$$

$$W^{*} = \begin{cases} W + \dfrac{1{,}25 \cdot t}{\pi}\left(1 + \ln\dfrac{4\pi W}{t}\right) ; & \text{for } \dfrac{W}{h} \leq \dfrac{1}{2\pi} \\[2ex] W + \dfrac{1{,}25 \cdot t}{\pi}\left(1 + \ln\dfrac{2h}{t}\right) ; & \text{for } \dfrac{W}{h} \geq \dfrac{1}{2\pi} \end{cases} \quad (5)$$

$$\varepsilon_{reff}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{reff}}{1 + G} \quad (6)$$

$$\text{where } G = 0{,}168(\varepsilon_r - 1) \cdot \left(\frac{f}{\text{GHz}}\right)^2 \cdot \left(\frac{h}{\text{mm}}\right)^2 \cdot \left(\frac{Z_L}{\Omega}\right)^{-\frac{3}{2}}$$

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junction the approximations are given in /3/ **equations (10) and (11)**. The reference plane displacement is taken from the centre lines' of the conductors. The characteristic impedance Z_1 is that of the through line and Z_2 is the impedance of the branch line. These impedances are determined with **equations (1) and (2)** or **diagrams 1 and 2** together with the relevant effective relative permittivity.

5. REFERENCES

/1/ Mehran, R.: Grundelemente des rechnergestützten Entwurfs von Mikrostreifenleitungs-Schaltungen
Verlag H. Wolff, Aachen

$$\frac{d}{h} = 0{,}434907\,\frac{(\varepsilon_{reff})^{0{,}81} + 0{,}26}{(\varepsilon_{reff})^{0{,}81} - 0{,}189} \cdot \frac{(\frac{W}{h})^{0{,}8544} + 0{,}236}{(\frac{W}{h})^{0{,}8544} + 0{,}87} \cdot M \quad ; \qquad (7)$$

$$\text{where} \quad M = \frac{(1-0{,}218e^{-7{,}5\frac{W}{h}})\,(1+\frac{0{,}5274}{(\varepsilon_{reff})^{0{,}9236}})\ \arctan(0{,}084(\frac{W}{h})^{K})}{1+0{,}0377(6-5\cdot e^{-0{,}036(\varepsilon_r-1)})\ \arctan(0{,}067(\frac{W}{h})^{1{,}456})} \quad ;$$

$$\text{where} \quad K = 1{,}9413\,(1+\frac{(\frac{W}{h})^{0{,}371}}{1+2{,}358\cdot\varepsilon_r})^{-1} \quad ;$$

$$b = W\,\frac{1-0{,}9\cos(\varphi/2)}{\sin(\varphi/2)} \quad ; \qquad (8a)$$

$$b = \frac{W}{\sqrt{2}} - d \qquad (8b)$$

with d from equation (7) and with width $\sqrt{2}$ W determined

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$$\Delta\ell = \frac{h}{2}\,(0{,}5 \cdot (\frac{W}{h})^{1{,}88} - \frac{0{,}45}{\sqrt{\varepsilon_r}} - 0{,}12) \qquad (9)$$

$$d_2 = 0{,}5\,D_1 - \frac{D_1 Z_2}{Z_1}(0{,}076 + 0{,}2(\frac{2D_1}{\lambda})^2 + 0{,}663\cdot e^{(-1{,}71\frac{Z_1}{Z_2})} - 0{,}172\,\ln\frac{Z_1}{Z_2}) \qquad (10)$$

$$d_1 = 0{,}05 D_2\,\frac{Z_1}{Z_2}\left(\frac{\sin(\frac{\pi}{2}\cdot\frac{2D_1}{\lambda}\cdot\frac{Z_1}{Z_2})}{\frac{\pi}{2}\cdot\frac{2D_1}{\lambda}\cdot\frac{Z_1}{Z_2}}\right)^2 \cdot (1-(\pi\frac{D_2-2d_2}{\lambda})^2) \qquad (11)$$

$$\text{where } D_1 = \frac{120\pi\Omega\cdot h}{\sqrt{\varepsilon_{reff1}}\cdot Z_1} \quad ; \qquad D_2 = \frac{120\pi\Omega\cdot h}{\sqrt{\varepsilon_{reff2}}\cdot Z_2}$$

$$\lambda = \frac{c_0}{\sqrt{\varepsilon_{reff1}}\cdot f}$$

/2/ Hoffmann, R. K.: Integrierte Mikrowellen-schaltungen
Springer Verlag, Berlin 1983

/3/ Hammerstad, E. O.: Equations for microstrip circuit design
Proceedings of the 5th EMC, 1975, p 268 - 272

/4/ Lieferfirma für RT/Duroid:
Mauritz GmbH & Co., Postfach 10 43 06, 2000 Hamburg 1

/5/ Lieferfirma für Di-Clad:
Municom, Postfach 12 10, 8217 Grassau