Calculating the Heights of Microwave Antennas in the Case of One Obstacle in the Far Field

Accommodate an obstruction when designing a microwave system

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The calculation of microwave antenna heights is becoming much more common in telecommunications world due to the intense growth of implementing digital microwave radios, not only in urban but also in country areas. So, due to the great importance of this subject it is presented here a simple formulation to perform this calculation in the case of one simple obstacle along the path (the obstacle is situated in the far-field region).

Here is a tutorial review of the design of a microwave path, which considers the factors of distance, frequency-dependent effects (Fresnel zone) and the height of an obstruction in the path, perhaps a building, tree or hill. The goal is to determine the necessary height of the antennas placed at each end of the path. Once the path equations have been developed, the procedures for creating a software program using them will be discussed.

Some useful definitions

Let us consider a path from one microwave station (site *A*) to another microwave station (site *B*). Next we have the definitions of the symbols used there.

H_A	Altitude, relating to the MSL (Mean Sea Level), of the ground level at site *A*.
h_A	Height (from the ground) of antenna port at site *A*.
H_B	Altitude, relating to the MSL, of the ground level at site *B*.
h_B	Height (from the ground) of antenna port at site *B*.
d_1	Distance between site *A* and the critical point (CP).
d_2	Distance between CP and site *B*.
d	Total distance between sites *A* and *B*; ($d = d_1 + d_2$).
H	Altitude, relating to the MSL, from the ground level at CP.
h_{cp}	Height of the obstacle (over the ground level) at CP (a building or a tree, for example).
Δh	Earthbulge at the critical point CP.
R_{F1}	Radius of the first Fresnel ellipsoid at CP.
F	Height clearance due to various measurement uncertainties.

Each of these physical entities should be expressed in the same unit of length (usually the meter in the SI System, or the foot in the BSWM (British System). The meter (m) and the foot (ft.) are related as follows.

1 ft = 0.3048 m (exactly)

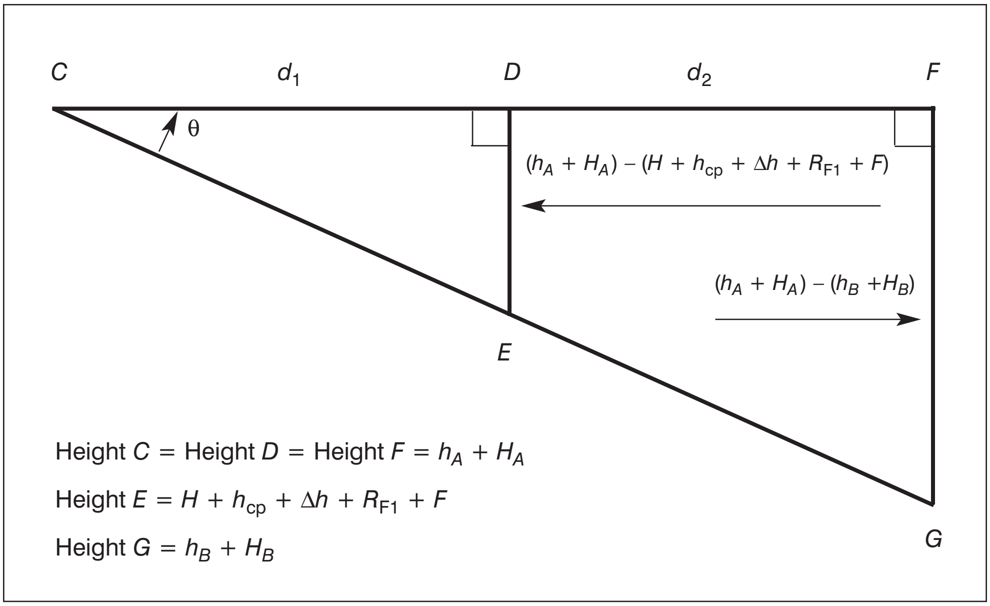
Normally the distances are given in kilometres (km) in the SI System or in terrestrial (statute) miles (mi) in the BSWM System. The metre, the kilometer, the foot and the statute mile are related as follows:

1 km = 1000 m
1 mi = 5280 ft = 1609.344 m (both exact)

The MSL (Mean Sea Level) is the major reference for measuring altitudes (it is the so called vertical datum or altimetric datum). Every country has its own vertical datum (or even datums). In Brazil, for example, the official altitudes are measured in terms of the vertical datum named Marégrafo de Imbituba (located in the state of Santa Catarina, south Brazil). An altitude relating to the MSL is normally said to be an "AMSL Altitude" (Above Mean Sea Level Altitude).

The port of a microwave parabolic antenna means the feeding interface of the antenna, that is to say, the interface between the antenna feedhorn and its feeding transmission line (like a coaxial cable or an elliptical, rectangular or circular waveguide).

A site means a place where we have a microwave station installed (or a terminal station or a repeater). A CP (critical point) is a physical obstacle for the direct propagation of the microwave radio signals (like a building, a tree or a hill).



▲ **Figure 1. Resolving triangles to develop path formulas.**

The earth bulge Δh of a certain physical obstacle is the altimetric correction to take into account not only the bending of microwave radio signals, due to the propagation through the tropospheric layers, but also Earth curvature. Mathematically we can write:

$$\Delta h = \frac{d_1 d_2}{2.74K} \tag{1}$$

where both d_1 and d_2 were previously defined and K is the propagation factor or EERF (Effective Earth Radius Factor — the coefficient of the apparent radius of the Earth). Distances d_1 and d_2 must be expressed in kilometers in order to have an earth bulge Δh expressed in meters (note the presence of the transforming numerical factor 12.75). K-factor is a scaling factor and is merely an adimensional one (that is to say, it is a ratio between two entities, both expressed in the same unities).

$$K = \frac{R}{a} \tag{2}$$

where R is the fictitious radius of the Earth (km); a is the actual mean radius of the Earth (km), approximately equal to 6370 km. Another possibility is the use of British units. In this case we have:

$$H_u = \frac{d_1 d_2}{1.5K} \tag{3}$$

In this formula the distances d, d_1 and d_2 must all be expressed in statute miles (mi) and the signal frequency in GHz. H_u will be given in feet due to the transforming numerical factor 1.5.

The radius R_{F1} of first Fresnel ellipsoid (at the critical point CP) is mathematically given by:

$$R_{F1} = 17.3\sqrt{\frac{d_1 d_2}{d f}} \tag{4}$$

where f is the microwave radio frequency. In this formula the distances d_1, d_2 and d must all be expressed in km and the signal frequency f must be expressed in GHz. Using km and GHz, R_{F1} will be given in meters.

Another possibility is the use of British units. In this case we have:

$$R_{F1} = 72.2\sqrt{\frac{d_1 d_2}{d f}} \tag{5}$$

In this formula the distances d, d_1 and d_2 must all be expressed in statute miles (mi) and the signal frequency in GHz. R_{F1} will be given in feet due to the transforming numerical factor 72.2.

At this moment we may experience a doubt. Normally we transmit a signal spread over a frequency band (not only a unique and well defined frequency). What is the frequency value we must use to calculate the ellipsoid radius? The most appropriate answer is the carrier frequency ($f = f_c$).

The clearance F, due to the uncertainties of the relief, is a positive increment over the measured height (h_{med})

to take into account possible measuring errors. If the prospecting survey is well done there is no necessity of considering clearance F over the measured obstacles.

Sometimes the clearance F is used when we want to include the possible growth of trees in the vicinity of the obstacle (eucalyptus farms are classical cases).

Formula development

Let there be right triangles CDE ($D = 90°$) and CFG ($F = 90°$). The heights of these five points are shown in Figure 1, where:

height C = height D = height $F = h_A + H_A$

height $E = H + h_{cp} + \Delta h + R_{F1} + F$

height $G = h_B + H_B$

Since all the entities are expressed in the same units let's calculate the tangent of the angle θ.

$$\tan(\theta) = \frac{(h_A + H_A) - (H + h_{cp} + \Delta h + R_{F1} + F)}{d_1} \qquad (6)$$

because segment DE is the superior height (D) minus the inferior height (E), that is to say:

$$\overline{DE} = (h_A + H_A) - (H + h_{cp} + \Delta h + R_{F1} + F)$$

In the CFG right triangle we have:

$$\tan(\theta) = \frac{(h_A + H_A) - (h_B + H_B)}{d_1 + d_2} \qquad (7)$$

Expliciting h_B (our unknown variable) as a function of all the other quantities, we have:

$$h_B = h_A + H_A - H_B - d\tan(\theta) \qquad (8)$$

substituting the previously calculated value for the tangent of the angle θ we finally have:

$$h_B = h_A + H_A - H_B - d\frac{(h_A + H_A) - (H + h_{cp} + \Delta h + R_{F1} + F)}{d_1} \qquad (9)$$

The height h_B can now be set as a function of all other quantities:

$$h_B = \text{Func}(h_A, H_A, H_B, d_1, d, H, h_{cp}, \Delta h, R_{F1}, F)$$

The previous formula is only valid if the obstacle is located in the far-field region of the two microwave parabolic antennas, that is to say:

$$d_{ff} = \frac{2D_2}{\lambda} \qquad (10)$$

where, λ is the signal operating wavelength (m); d_{ff} is the far-field distance (km); D is the diameter of the microwave parabolic antenna (m).

Approach for the calculations

The quantities H_A, H, h_{cp} and H_B must be measured in the field or read from topographic maps. The distances d_1, d_2 and d can be measured in the field itself (by distance meters or telurometers), measured using topographic maps or calculated from their own geodetic coordinates (sites A and B and the obstacle CP).

The earthbulge Δh and the first Fresnel ellipsoid radius R_{F1} are calculated using previous presented formulae. The clearance F must be selected according to the obstacle specific situation (see the previous remarks).

I have made a computer program for the easy evaluation of microwave antenna heights. The program has three main modules: an input module, a calculating module and an output module (for a printer, a video monitor or both). We can also save a calculation sheet for further consulting or working.

Input variables are: H_A, H_B, d_1, d, H, h_{cp}, f (frequency), F (clearance) and the pair K-factor/desobstruction. The software calculates the distance d_2 ($d_2 = d - d_1$), Δh and R_{F1}

This software was written using Quick BASIC (by Microsoft Corporation), and presently it is working under its executable version (an EXE-type archive).

Since we have all these entities previously defined, the formula in (6) must be put inside a finite loop (FOR-TO-NEXT type repetition structure). The calculation began with a "randomic" initial value for h_A (generally zero metres) and calculates its corresponding h_B value.

The calculation goes on using constant positive increments over h_A values (frequently this step is 5 metres) and the corresponding values of h_B (our output variable). The platform increment is commonly 5 metres because in long-distance, high-capacity microwave radio systems the towers have platforms generally spaced 5 metres from others. Not only to facilitate installation but also maintenance, the microwave parabolic antennas must always be installed wherever there are platforms on towers.

Height h_A receives an initial value. Using equation (9) the software calculates the second element of the pair (h_A, h_B) generating a table of theoretical antenna allowed pairs.

Normally a good software asks two values for the propagation factor K. The first one is the so called median-K (= K 50%) and the second one is the minimum-K (for instance, K 0.1% or K 0.01%). The software also asks for the desired per cent desobstruction for the first Fresnel zone radius. For median-K normally we use 100% desobstruction (one hundred per cent of first Fresnel zone radius free of obstacles) and for minimum-K normally we use 60% desobstruction (sixty per cent of first Fresnel zone radius free of obstacles). In some special cases we use a desobstruction of only 30% for the minimum-K.

When we use two different values for the propagation factor K (and their corresponding per cent desobstructions; K_1, K_2, $desob_1$ and $desob_2$) the software can print two tables (one table for each value of K) or just only one table (the worst-case table).

The software permits us, obviously, to work with several obstacles and not just only one (to be more evident, the obstacle-vector has 10 positions, but this number can be easily altered in the BASIC source code).

Since we must be as generic as possible, the user must always enter the platform increment (without entering a new value the program considers its default value as 5 meters).

When an error has already occurred

One time, in a certain prospecting survey report, there was an inversion of the numerator sign in equation (6). For this reason, a serious error has ocurred. Here it is the demonstration of the way (in fact, the wrong way).

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin(\theta)}{\cos(\theta)} = -\tan(\theta)$$

and thus,

$$\tan(\theta)_{right} = Y$$

$$\tan(\theta)_{error} = -Y$$

then,

$$\tan(\theta)_{right} = -\tan(\theta)_{error}$$

$$h_{B\ right} = h_A + H_A - H_B - d \times \tan(\theta)_{right}$$

$$h_{B\ error} = h_A + H_A - H_B - d \times \tan(\theta)_{error}$$

Subtracting one equation from the other one we finally have:

$$h_{B\ error} = h_{B\ right} + 2d \times \tan(\theta)_{right}$$

The change of the tangent sign (in equation 6) has generated an error equals to:

$$h_{B\ error} - h_{B\ right} = \text{Error}$$

$$\text{Error} = 2d \times \tan(\theta)_{right} \quad (11)$$

The error magnitude is somewhat large. Let it be a typical microwave radiolink stated as follows.

d = 10 km		K = 4/3 ~1.333
d_1 = 4 km		d_2 = 6 km
h_A = 750 m	— site A —	H_A = 45 m
h_B = 230 m	— site B —	H_B = 30 m
H = 550 m	— CP —	h_{cp} = 20 m
Δ_h = 1.4 m		R_{F1} = 8.1 m
F = 1.5 m		f = 10.98 GHz

This center frequency (f = 10.98 GHz) belongs to the 11 GHz frequency plan (it is the geometric mean between the two frequencies of channel number one, that is to say, f_1 = 10.715 GHz and f_2 = 11.245 GHz).

So, the trigonometric parameter tan(θ) is equal to:

$$\tan(\theta) = 0.0535$$

The error (expressed in kilometres) is equal to:

Error = 2 × (10 km) × (0.0535) = 1.07 km
(about 0.665 statute mile)

Summary

The development of equations for determination of antenna height in a microwave system has been shown here, including allowance for an obstruction located in the path. It is hoped that this review will make the design of such systems easier.

Author information

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