Peter Gerber, HB 9 BNI

The Doppler Effect over Radio Links using Active or Passive Reflectors

This article describes the physical fundamentals necessary for an understanding of the Doppler Effect together with the necessary formulae. It also presents a method of calculating the influence of the Doppler Effect on contacts made via paths where both active and passive reflectors are involved. This calls for a sub-program which permits the calculation of the reflector co-ordinates to any desired point in time.

1. PRINCIPLES

In the case of a communication path which is continually varying in distance between transmitter and receiver, the received frequency is not the same frequency as that which was transmitted. This frequency alteration is called the Doppler Effect after its discovery by the physicist Doppler. A similar effect is well known at audio frequencies when, for example, a railway locomotive with a continuous horn is passing an observer. The pitch of the horn always sounds higher to the observer when the locomotive is approaching than when it is departing. For this acoustical phenomenon the physical theory is extremely complicated as it must always be decided what is moving relative to whom taking into consideration the sender, the receiver and the communication medium.

As electromagnetic waves exhibit no measurable relative movement to the medium through which it passes (experiments by Michelson and Einstein's theory of relativity) at least one variable may be eliminated, and that leaves only the relative movements of transmitter and receiver. When considering the case where relative speeds approaching light is concerned, the theory is also very complicated such as the velocity components in the line-of-sight as also the transverse components give rise to a Doppler Effect. With small relative velocities, and that includes all amateur contact paths, only the line-of-sight component V_s plays a part in the Doppler

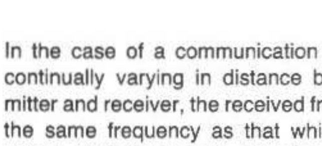
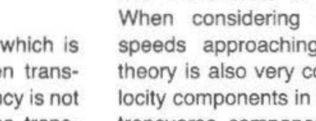


Fig. 1



S = Sender
R = Receiver

Fig. 2

Effect (**fig. 1 and fig. 2**). In practice, the case of transmitter and receiver moving away from each other is so small, and occurs so seldom for the amateur anyway (see example below). The Doppler Effect, however, plays a very prominent role in paths, using either active or passive reflectors, made in space via satellites and EME (4).

2. PHYSICAL FUNDAMENTALS

The transmitter, of frequency f, moving with a velocity v directly to a receiver (**fig. 2**) will be measured at the receiver as: –

$$f_r = f\,(1 - v / c) \qquad (1)$$

where v is considered a negative quantity when the sender S and the receiver R approach each other. The received frequency f_r is then higher than the transmitted frequency f.

Example:
The sender, with a frequency of 145 MHz, moves towards the receiver at a speed of 120 km/h. What will be the received frequency?

$v = -120$ km/h $= -33.33$ m/sec.
$f = 145$ MHz $= 145 \times 10^6$ 1/sec.
$c = 300 \times 10^6$ m/sec.

$$f_r = 145'000'000 \times (1 + \frac{33.33}{300'000'000})$$

$$= 145 \times 10^6 \times 1.000000111 = 145'000'016 \text{ Hz}$$

The received frequency f_r, is therefore 16 Hz higher than the send frequency which, normally, has no practical consequence. Transposing the eq. 1 the following quantities may be obtained:

Change in frequency $df = f\,(-v / c)$
Change in wavelength $dl = l\,(v / c)$

The point to watch is that all quantities are presented to the equation in the same units i.e. metres, metres per second, hertz. Also, that the velocity sign is negative when the transmitter approaches the receiver. The main problem, when calculating the Doppler Effect on signals via reflectors (active or passive), is that of determining the velocity of sender and receiver in the direct path. The sender, receiver and the reflector are mostly moving in complicated relative orbits in space. The sender on earth moves with the earth, the satellite in a canted ellipse, with variable speed and direction and the receiver, again, with the earth. Active satellites have the additional consideration of the alteration in frequency due to frequency changing processes within the satellites equipment. These "conversion functions" must also be taken into account when calculating the received frequency.

3. CALCULATIONS

The calculated received frequency of a signal, transmitted via active or passive reflectors, is arrived at by using the following steps:

- Transmitted frequency f_{u1} (uplink 1)
- Uplink Doppler Effect
- Received frequency f_{u2} (uplink 2)
- Transit function
- Transmitted frequency f_{d1} (downlink 1)
- Downlink Doppler Effect
- Received frequency f_{d2} (downlink 2)

If the transmitter and the receiver are both located at the same place then the same relative velocity is used for the Doppler Effect calculation. If a passive reflector is involved, the transit function is simply $f_{u2} = f_{d1}$.

It has already been mentioned that the chief problem lies in the calculation of the relative velocities of the participating stations in the direct point to point path. The radio amateur OK 1 DAT gives, for example, in (1) the following formula for the Doppler Effect over an EME link (but without considering the influence of the elliptical nature of the moon's orbit):

$$df = -\frac{2\,\pi\,6370 \cos BE \sin AH \cos DE}{24 \times 3.6\,\lambda\,(1 + 0.034 \cos AH)}$$

When calculating the effect over satellites, the formula is even more complicated because the elliptical orbit must be taken into account.

Using a small computer for calculating the orbit of the transponder, a simple procedure may be employed.

The principle lies in the fact that the distance from the sender/transponder and the receiver/ transponder can be arrived at relatively easily. Should the distance to two different, but not too far apart in the time frame, points be calculated, the average line-of-sight velocity in this time frame may be taken, namely the velocity component in the most direct path.

The distance calculation is carried out simply by converting the main polar co-ordinates to the Cartesian form and then using three-dimensional Pythagoras. The necessary formulae for this are given as follows:

Conversion from polar to Cartesian co-ordinates given that a longtitude information is L
a latitude information is B
a radius information is R

The Cartesian co-ordinates are X, Y and Z:

$$X = R \cos B \cos L$$
$$Y = R \cos B \sin L$$
$$Z = R \sin B$$

When these co-ordinates have been calculated for both transmitter and receiver and the Cartesian co-ordinates subtracted from each other viz.

$$DX = X1 - X2$$
$$DY = Y1 - Y2$$
$$DZ = Z1 - Z2$$

then the instantaneous distance is as follows: –

$$\text{Distance } E = \sqrt{(DX \times DX + DY \times DY + DZ \times DZ)}$$

A point to watch is that the transmitter and the receiver (= transponder) should use polar co-ordinates which are compatible with each other, as for example, those given in **table 1**.

A general procedure for the calculating process

EARTH			TRANSPONDER		
Longtitude	**Latitude**	**Radius**	**Longtitude**	**Latitude**	**Radius**
longtitude	latitude	earth radius	longtitude subsatel-lite point	latitude subsatel-lite point	Distance from earth's centre
local startime	latitude	earth radius	right ascension	declen-sion	geocentric distance
longtitude	latitude	earth radius	hour angle (GHA)	declen-sion	geocentric distance
0	latitude	earth radius	local hour angle	declen-sion	geocentric distance

Table 1: Mutually matching polar co-ordinates

may be given as follows:

- Calculate the polar co-ordinates of the QTH and the transponder.
- Calculate distance E 1
- Convert QTH into Cartesian co-ordinates
- Convert transponder into Cartesian co-ordinates
- Subtract the Cartesian co-ordinates
- Use Pythagoras to find the distance
- Calculate the polar co-ordinates of the QTH and the transponder for a later point in time (e.g. after 60 s)
- Calculate the difference in the distances and divide by the time difference to obtain the velocity
- Calculate the Doppler Effect of the link

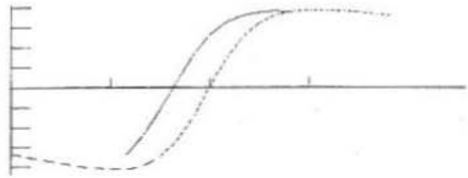
When a passive transponder is used, the Doppler shift is multiplied by a factor of 2. Active transponders require separate calculations for uplink, transit through the transponder, and downlink.

4. APPLICATION EXAMPLES

The Doppler Effect for an EME contact via the AMSAT-OSCAR 10 was calculated using the above mentioned formulae as a base.

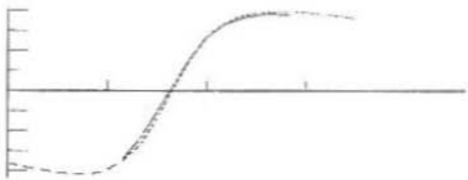
The calculation for the EME path was made using both the given formulae and that given by OK 1 DAT. The results on the whole, were compatible, the maximum difference amounted to 10 Hz for a transmitter frequency of 144 MHz (Doppler shift + 350 Hz to − 350 Hz). This discrepancy arose from the influence of the elliptical nature of the moon's orbit which was approximated in the OK 1 DAT calculation.

There was no alternative method available of calculating the effect when assessing the OSCAR 10 link. The Doppler Effect on this link, on the other hand, amounted to over 3000 Hz. This shift can be easily measured with amateur equipment. Suitable measurements were carried out and evaluated (**fig. 3 and fig. 4**). The calculations are so exact that, even in modest circumstances, the orbit data of OSCAR 10 may be evaluated.



DATA: TIME SCALE: 9.3Ø UT
ARG.PERIG. 40 RAAN 121
TIME Ø MIN FREQ.SHIFT 14ØØ HZ
EPOCHE G: 2776.ØØ899

Fig. 3: Comparison between measured (solid line) and calculated (dotted line) Doppler shift using OSCAR 10 before adjustment. The satellite, according to the calculation, is a little premature.



DATA: TIME SCALE: 9.3Ø UT
ARG.PERIG. 40 RAAN 121
TIME -1Ø MIN FREQ.SHIFT 14ØØ HZ
EPOCHE G: 2776.ØØ2Ø5

Fig. 4: The same data following the matching of the satellite's data (time shift about 10 min.)

5. REFERENCES

(1) OK 1 DAT, ohne Titel, handelt vom Doppler-Effekt bei EME, DUBUS Buch # 2, p 28 ff

(2) Autorenteam:
Meyers Handbuch über das Weltall,
Mannheim, 1974

(3) O. Montenbruck:
Grundlagen der Ephemeridenrechnung,
Verlag Sterne und Weltraum, München, 1984

(4) Gerber, P., HB 9 BNI:
EME für jedermann?
"old man" 9/1986, Seite 23 - 26