RF Oscillator Analysis and Design by the Loop Gain Method

Here is another perspective on the design of oscillators

By Noël Boutin
Université de Sherbrooke

This tutorial paper presents a formal method of RF oscillator analysis and design involving the loop gain. This method must not be confused with the open-loop gain method, which is very often presented in literature as the loop gain method. We will show that despite the fact we can extract the right values of the frequency of oscillation and the startup condition from both methods, the loop gain is the only method that always gives us the right value of the phase slope, or the effective loaded Q, at the operating point.

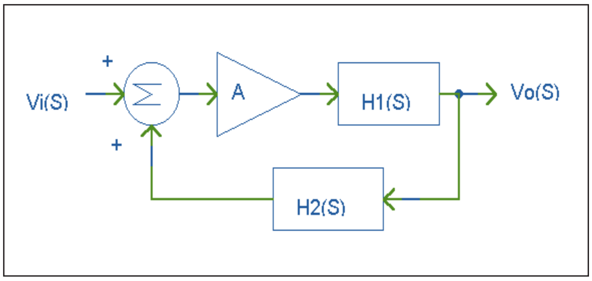
An oscillator as an amplifier with feedback

The loop gain method considers an oscillator as an amplifier with feedback. Figure 1 shows the classical block diagram of such an amplifier. This diagram is presented in almost all electronic textbooks.

It is generally accepted (but not necessarily true) that the practical amplifier is represented by an ideal amplifier with gain A, cascaded with a transfer function characterizing its frequency response. The output signal is returned via an external transfer function, and is then added with the input signal. The result of this summation is applied to the amplifier input. Without the feedback transfer function, the transfer function of the amplifier is given by equation 1.

$$\left(\frac{V_0(S)}{V_i(S)}\right)_{WOF} = A \cdot H_1(S) \tag{1}$$

With the presence of the feedback transfer function, the overall transfer function of the feedback amplifier is given by equation 2.



▲ **Figure 1. Block diagram of an amplifier with feedback.**

$$\left(\frac{V_0(S)}{V_i(S)}\right)_{WF} = \frac{A \cdot H_1(S)}{1 - A \cdot H_1(S) \cdot H_2(S)} = \frac{\left(\frac{V_0(S)}{V_i(S)}\right)_{WOF}}{1 - \text{Loop gain}} \tag{2}$$

The transfer function is qualified as "the loop gain." It corresponds to the gain that a signal injected at some point within the loop would encounter after one complete clockwise trip around the loop.

The characteristic equation of the overall transfer function of the feedback amplifier is given by equation 3.

$$1 - A \cdot H_1(S) \cdot H_2(S) = 0 \tag{3}$$

The roots, or the zeros, of the characteristic equation are the poles of the overall transfer function of the feedback amplifier.

An oscillator is obtained from the feedback amplifier when an output signal is present in the absence of an external input signal. This is possible only if the denominator of equation 2 equals zero; that is, the loop gain is equal to unity for some values of the complex frequency S. When this happens for only two conjugate imaginary values of S, a constant amplitude sinewave output frequency signal is obtained. This condition is known as the Barkhausen criteria and can be mathematically expressed as in equation 4.

$$\begin{aligned} &\left|A \cdot H_1(\pm j\omega_0) \cdot H_2(\pm j\omega_0)\right| = 1 \\ &\text{Phase}\left[A \cdot H_1(\pm j\omega_0) \cdot H_2(\pm j\omega_0)\right] = 0 \end{aligned} \quad (4)$$

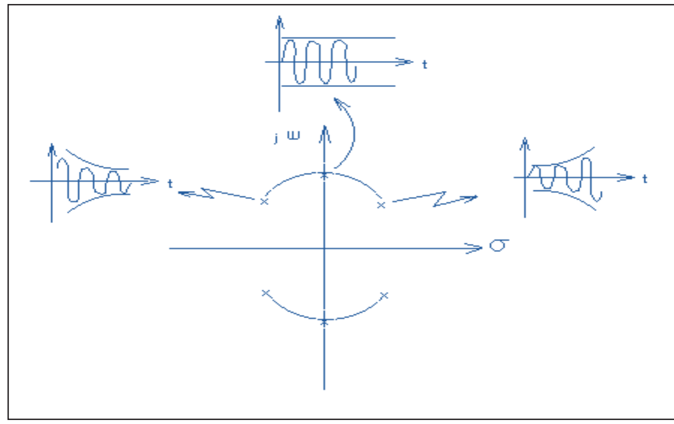
This criteria makes sense intuitively, since it states that, at the frequency of oscillation ω_0, the signal must go around the loop with no attenuation and no phase shift.

When the Barkhausen criteria is met, the two conjugate poles of the overall transfer function (equation 2) are located on the imaginary axis. Any departure from that position will lead to an increase or a decrease of the amplitude of the output sinewave signal in the time domain. This kind of change would occur according to how the conjugate poles move in the right or in the left half plane of the axis, as shown in Figure 2. In practice, the equilibrium point cannot be reached instantaneously without violating some physical laws. The sinewave cannot start at full amplitude instantaneously after the power supply is turned on. With other practical considerations such as component tolerances, the non-linearity of the amplifier and the desire to oscillate at a given amplitude, the design of the circuit must be such that, at start up, the poles are located in the right half plane, but not too far from the axis. This is achievable with a small signal loop gain greater than unity. As the sinewave amplitude builds up, at least one parameter of the loop gain must change its value in such a way that the two complex conjugate poles migrate in the direction of the axis. That parameter must then reach that axis for the desired steady state sinewave amplitude value. At this point, the large signal loop gain equals unity and the Barkhausen criteria is met.

Most of the time, the loop gain value is dependent on the amplitude value of the sinewave present at the active device input terminals. This is because of the intrinsic nonlinear nature of the active device itself. In this case, the oscillator circuit is qualified as "self-limiting." Suffice it to say that an active device powered by a finite value DC supply cannot linearly amplify a signal irrespective of its amplitude value. Therefore, the effective gain of a practical amplifier always decreases as the amplitude of the input signal increases.

The difference between the loop gain and the open loop gain

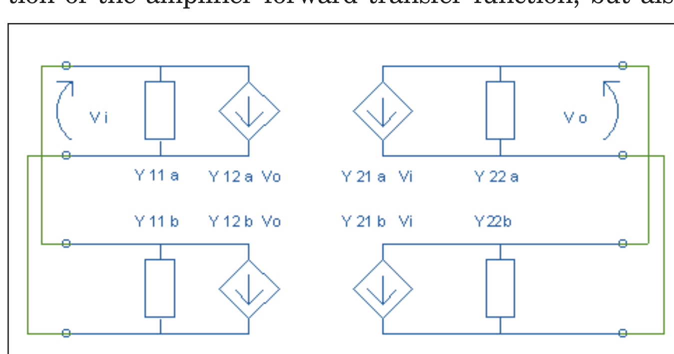
The feedback amplifier illustrated in Figure 1 can be implemented in four different configurations known as the shunt-shunt, series-shunt, shunt-series and series-series feedback, depending on whether a voltage or a current is added to the input signal or subtracted from the output signal. For convenience, we will consider only the shunt-shunt feedback, however, the analysis method presented is completely general and can be applied to the other feedback configurations with the proper adaptation. It is worth noting that a given oscillator circuit can often be regarded in more than one configuration.



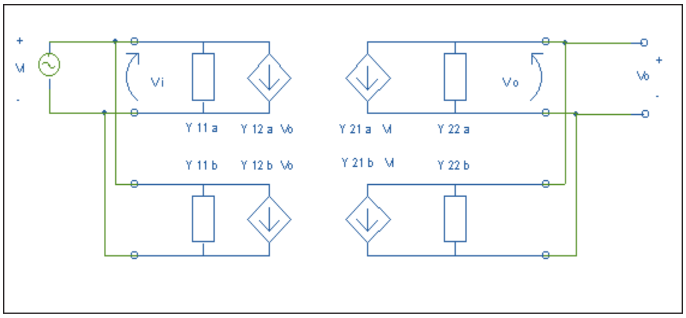
▲ **Figure 2. Frequency domain root locus and the corresponding time domain response.**

As the amplifier and the feedback network are connected together in a shunt-shunt arrangement, it is convenient to model both of them as two-port y-parameter networks where the indexes a and b refer respectively to the amplifier and the feedback network, as shown in Figure 3.

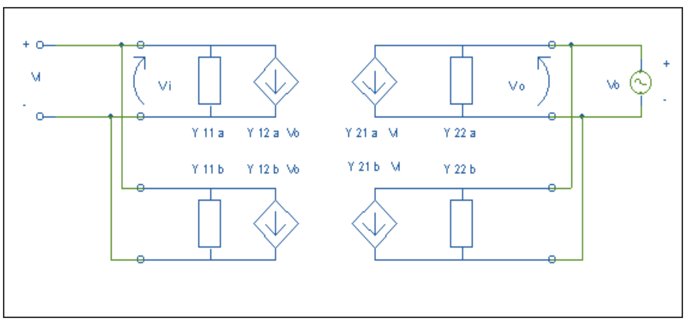
In order to correctly evaluate the loop gain value of the circuit model shown in Figure 3, one must first establish the right correspondence between that model and the block diagram shown in Figure 1. Two implicit fundamental hypotheses that are often forgotten were made when deriving equations 1 and 2. The first one is that the transfer function is unilateral from left to right, and represents the total forward transmission transfer function from the input to the output. It is not only the contribution of the amplifier forward transfer function, but also



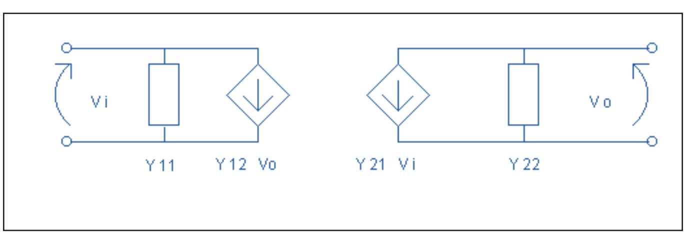
▲ **Figure 3. As oscillator circuit modeled as a shunt-shunt connection of two-port networks.**



▲ **Figure 4. Setup used to evaluate $A \times H_1(S)$.**



▲ **Figure 5. Setup used to evaluate $H_2(S)$.**



▲ **Figure 6. Two port network model of an amplifier.**

of the passive external network (which corresponds to the reverse transfer function of the passive network if one considers the right terminals as the input of the passive network). The second hypothesis is that the transfer function is also unilateral from right to left, and represents the total reverse transmission transfer function from the output to the input. It is not only the passive transfer function but also the contribution of the reverse transfer function of the amplifier.

The forward transfer function of the circuit shown in Figure 3 is obtained by connecting an ideal independent voltage source at the left input port terminals (Figure 4) and evaluating the output voltage response at the right output port terminals. That ideal voltage source neutralizes the effect produced by the reverse controlled sources of both networks. Doing so, we find

$$A \cdot H_1(S) = \frac{V_0(S)}{V_i(S)} = -\frac{\left(y_{21_a} + y_{21_b}\right)}{\left(y_{22_a} + y_{22_b}\right)} = \frac{y_{21_{Total}}}{y_{22_{Total}}} \tag{5}$$

In a similar way, the reverse transfer function is obtained by connecting an ideal independent voltage source at the right output port terminals, as shown in Figure 5, and evaluating the input voltage response at the left input port terminals. That ideal voltage source neutralizes the effect produced by the forward controlled sources of both networks. Doing so, we find:

$$H_2(S) = \frac{V_i(S)}{V_0(S)} = -\frac{\left(y_{12_a} + y_{12_b}\right)}{\left(y_{11_a} + y_{11_b}\right)} = \frac{y_{12_{Total}}}{y_{11_{Total}}} \tag{6}$$

The results obtained in equations 5 and 6 are of first importance. They tell us that it is generally false to assume that the transfer function refers to the active part of the oscillator, and that it refers solely to the passive feedback frequency selective network.

The loop gain is given by the product of equation 5 and 6, which is:

$$A \cdot H_1(S) \cdot H_2(S) = \frac{\left(y_{12_a} + y_{12_b}\right)\left(y_{21_a} + y_{21_b}\right)}{\left(y_{11_a} + y_{11_b}\right)\left(y_{22_a} + y_{22_b}\right)} = \frac{y_{12_{Total}} \; y_{21_{Total}}}{y_{11_{Total}} \; y_{22_{Total}}} \tag{7}$$

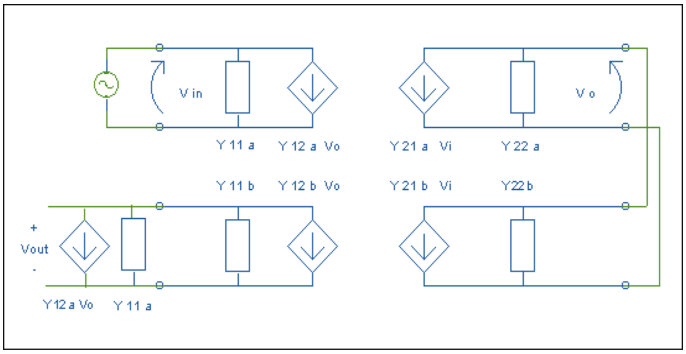
This result should not be a surprise to those familiar with the analysis of nonunilateral amplifiers or amplifiers with feedback. As an example, let us discuss the two port network amplifier shown in Figure 6. Without feedback,expressed as $y_{12} = 0$, the input admittance y_{in} is equal to y_{11}. With (internal or external) feedback, the input admittance is given by:

$$y_{in} = \frac{I_i}{V_i} = y_{11} + \frac{y_{12}V_0}{V_i} = y_{11} + \frac{y_{12}}{V_i}\left(\frac{-y_{21}V_i}{y_{22}}\right) = y_{11} - \frac{y_{12}y_{21}}{y_{22}} = y_{11}\left(1 - \frac{y_{12}y_{21}}{y_{11}y_{22}}\right) \tag{8}$$

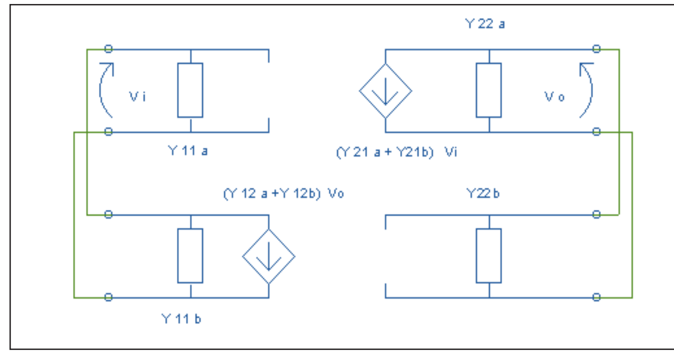
In order to show the effect of feedback upon the input (or the output) admittance of an amplifier, the result is also often written as:

$$y_{in} = y_{11} \cdot (1 - \text{Loop gain}) \tag{9}$$

Comparing equations 7, 8 and 9, one can conclude that we are continually discussing the loop gain, whether the circuit is an amplifier or an oscillator. This may explain why oscillators and feedback amplifiers are almost always simultaneously discussed in textbooks. It is puzzling, however, to observe that the procedure shown in almost every publication dealing with oscillators is the evaluation of the open loop gain instead of the



▲ **Figure 7. Open loop gain evaluation circuit.**



▲ **Figure 8. Rearrangement of the Figure 3 oscillator model for open loop gain circulation.**

evaluation of the loop gain already found in equation 7 or 8. It is said that in order to find the right value of the open loop gain, one must open the loop at an appropriate point (generally the input of the amplifier), inject an excitation and, in order not to change the loading effect, connect an impedance equal to the input impedance of the amplifier to the other open terminal. Doing so with the oscillator circuit shown in Figure 3, one gets the open loop circuit shown in Figure 7. The open loop gain is given by:

$$\frac{V_{out}}{V_{in}} = \frac{\left(y_{12a} + y_{12_b}\right) y_{21_a}}{\left(y_{11_a} + y_{11_b}\right)\left(y_{22_a} + y_{22_b}\right) - \left(y_{12_a} + y_{12_b}\right) \cdot y_{21_b}} \qquad (10)$$

Comparing that value of the open loop gain with the loop gain given in equation 8, one may conclude that the open loop gain is generally not equal to the loop gain, even if the loading effect is taken into account. A sufficient (but not necessary) condition for the equality of the open loop gain and the loop gain is that, before opening the loop, the oscillator circuit model must be rearranged as a parallel connection of two unilateral four port networks, as shown in Figure 8.

Taking into account the loading effect when opening the loop, it is easy to verify that the open loop gain of the circuit model shown in Figure 8 is equal to the loop gain given in equation 8. Other situations exist where the open loop gain is equal to the loop gain. One such situation is when one of the two port networks modeling the amplifier or the passive feedback network has an ideal controlled voltage source directly connected across its output terminals, defined here as the right terminals for the amplifier and the left terminals for the passive network. This may happen at low frequencies when modeling an op-amp or a transformer as an ideal device, but it is not at all realistic at RF.

In spite of the fact that the open loop gain value given in equation 12 is not equal to the loop gain value given in equation 8, both of them, derived from the same topology, lead to the same characteristic equation given by equation 11:

$$\begin{aligned} &1 - (\text{open loop gain}) = 1 - (\text{loop gain}) = \\ &\left(y_{11_a} + y_{11_b}\right)\left(y_{22_a} + y_{22_b}\right) - \left(y_{12_a} + y_{12_b}\right)\left(y_{21_a} + y_{21_b}\right) = 0 \end{aligned} \qquad (11)$$

One may argue that if they both give the same characteristic equation, then they must both give the right values of oscillation frequency and startup condition. This is true, but only in the above condition. As we will show in the next section, working with the open loop gain instead of the loop gain may lead to an incorrect conclusion regarding the phase slope or the loaded Q value at the operating point.

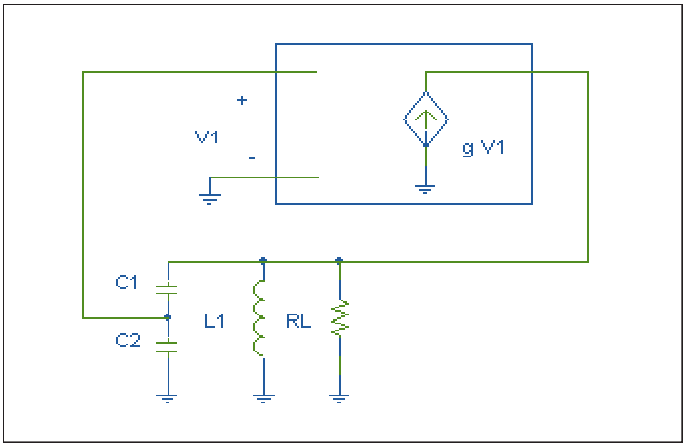
The case of an oscillator with an ideal unilateral amplifier

Let us consider the Colpitts oscillator shown in Figure 9. For the purpose of our discussion, we assume that the transconductance amplifier is ideal in the sense that its input impedance, output impedance and bandwidth are infinite. (This case may also include an unilateral amplifier with capacitive input taken into account within, or an unilateral amplifier with resistive output taken into account within).

As the input impedance is infinite, there is no loading and the loop can be open at the amplifier input terminals without any consideration. Doing so leads to the following open loop gain value:

$$\text{Open loop gain} = \left(\frac{C_1}{C_1 + C_2}\right) \cdot g \cdot \frac{1}{\left[S\left(\frac{C_1 C_2}{C_1 + C_2}\right) + \frac{1}{SL} + \frac{1}{R_L}\right]} \qquad (12)$$

The last term is the only frequency-dependent term, which can be recognized as the transfer function of a parallel RLC network. The open loop gain becomes purely real when the capacitive contribution is equal but opposite in sign to the inductive contribution. That hap-



▲ **Figure 9. Colpitts oscillator with an ideal transconductance amplifier.**

pens at a frequency given by:

$$\omega_0^2 = \frac{1}{L\left(\frac{C_1 C_2}{C_1 + C_2}\right)} \tag{13}$$

This value is independent of the g value. At that frequency, the open loop gain equals unity for the amplitude value of the sinewave for which the transconductance g becomes equal to:

$$g = \left(\frac{C_1 + C_2}{C_1}\right)\frac{1}{R_L} \tag{14}$$

In a steady state, the operating point on the open loop gain transfer function is located at the maximum amplitude (equal to unity) and at the maximum phase slope given by (refer to the box with useful mathematical relations at the end of the article):

$$\left.\frac{d\varphi}{d\omega}\right|_{\omega=\omega_0} = -2R_L\left(\frac{C_1 C_2}{C_1 + C_2}\right) = -2\frac{Q_L}{\omega_0} \tag{15}$$

with

$$Q_L \equiv -2R_L \cdot \omega_0\left(\frac{C_1 C_2}{C_1 + C_2}\right) \tag{16}$$

Let us recall here that the phase slope value indicates the frequency stability of an oscillator regarding any non-intentional variation in the value of a parameter related to the frequency of such an oscillator. The steeper the slope, the greater the parameter variation must be in order to produce a given change in frequency.

Now, lets consider the loop gain. Comparing Figure 9 with Figure 3, and evaluating the y-parameters of each one of the four port networks, one gets:

$$\begin{array}{ll} y_{11_a} = 0 & y_{11_b} = S(C_1 + C_2) \\ y_{12_a} = 0 & y_{12_b} = -SC_1 \\ y_{21_a} = -g & y_{21_b} = -SC_1 \\ y_{22_a} = 0 & y_{22_b} = SC_1 + \frac{1}{SL} + \frac{1}{R_L} \end{array} \tag{17}$$

Reporting those values of y-parameters in equation 7, one gets the following loop gain results:

$$\text{Loop gain} = \frac{C_1}{(C_1 + C_2)} g \frac{\left(1 + S\frac{C_1}{g}\right)}{SC_1 + \frac{1}{SL} + \frac{1}{R_L}} \tag{18}$$

There are two frequency dependent terms. The numerator term,

$$\left(1 + S\frac{C_1}{g}\right)$$

introduces a phase shift varying from 0° C to +90° C, whereas the denominator term,

$$\left[SC_1 + \frac{1}{SL} + \frac{1}{R_L}\right]$$

introduces a phase shift varying from +90 C° to -90° C. When both phase shifts are equal but opposite in sign, the total phase shift of the loop gain transfer function equals zero. This happens for a frequency given by (refer to box):

$$\omega_0^2 = \frac{1}{LC_1\left[1 - \frac{1}{gR_L}\right]} \tag{19}$$

It is worth noting that the zero phase shift frequency is dependent on the g value. The loop gain equals unity for the amplitude value of the sinewave for which the transconductance g becomes equal to (refer to box) :

$$g = \left(\frac{C_1 + C_2}{C_1}\right)\frac{1}{R_L} \tag{20}$$

which is the same value found in equation 14 with the open loop gain. Reporting that value of g in equation 19, one gets the same value of oscillation frequency as that found in equation 13 with the open loop gain. However, equation 18 tells us that in the steady state, the operating point on the loop gain transfer function is not necessarily located at or near the maximum amplitude and maximum phase slope. Incidentally, a lengthy but straightforward calculation of the loop gain transfer function phase slope at the steady state operating point, given by equation 20 and equation 13, leads to the following value (refer to box):

$$\left.\frac{d\varphi}{d\omega}\right|_{\omega=\omega_0} = \frac{-\dfrac{2\,Q_L}{\omega_0}}{1+\left(\dfrac{\omega_0}{g\,/\,C_1}\right)^2} \qquad (21)$$

where Q_L is given, as before, by equation 16. The slope value, found in equation 21 with the loop gain, is always smaller than the one found in equation 15 with the open loop gain. Therefore, if one designs such an oscillator using the open loop gain method instead of the loop gain method and does not choose the value where the frequency *(g/C_1)* is five to ten times the value of ω_0, the frequency stability obtained in practice will be poorer than the theoretical one. Now, let us consider a situation for which the open loop gain method predicts poorer performances than the those prevailing in reality.

The case of an oscillator with a unilateral amplifier with phase shift

The RF Colpitts oscillator model, shown in Figure 9, can be more representative of a real circuit if we take into account the phase shift introduced by a practical amplifier by substituting the real transconductance g with a complex one. Assuming in first approximation that the imaginary part varies linearly with frequency in the vicinity of the oscillation frequency, and that the phase shift introduced by the amplifier is negative, g becomes (g-SC_g). Doing so, the open loop gain and the loop gains respectively given in equation 12 and equation 18 become:

$$\text{Open loop gain} = \left(\frac{C_1}{C_1+C_2}\right)\cdot g \cdot \frac{\left(1-S\dfrac{C_g}{g}\right)}{\left[S\left(\dfrac{C_1C_2}{C_1+C_2}\right)+\dfrac{1}{SL}+\dfrac{1}{R_L}\right]} \qquad (22)$$

$$\text{Loop gain} = \frac{C_1}{(C_1+C_2)}\,g\,\frac{\left(1+S\dfrac{C_1-C_g}{g}\right)}{\left[SC_1+\dfrac{1}{SL}+\dfrac{1}{R_L}\right]} \qquad (23)$$

From equation 22 and equation 23, we can draw the following conclusions. On one hand, designing such an oscillator with the open loop gain method lets one erroneously believe from equation 22 that the negative phase shift introduced by the amplifier is detrimental. This negative phase shift must be compensated for by the band pass with a point of operation that must shift from the maximum zero phase shift slope at a frequency given by:

$$\omega^2 = \frac{1}{L\left(\dfrac{C_1C_2}{C_1+C_2}\right)} \qquad (24)$$

to a less steeper slope at a lower frequency given by (refer to box):

$$\omega_0^2 = \frac{1}{L\left(\dfrac{C_1\left(C_g+C_2\right)}{C_1+C_2}\right)} \qquad (25)$$

when the open loop gain equals unity, shown as (refer to box):

$$g = \left(\frac{C_1+C_2}{C_1}\right)\frac{1}{R_L} \qquad (26)$$

This conclusion would be confirmed experimentally or by simulation if one measures or simulates the open loop transfer function.

On the other hand, designing such an oscillator with the loop gain method lets one correctly conclude from equation 23 that a negative phase shift introduced by the amplifier can be completely compensated for by a proper choice of the value of C_1, that is $C_1 \equiv C_g$. Doing so lets the band pass operate at its maximum zero degree phase slope at a frequency given by:

$$\omega_0^2 = \frac{1}{LC_1} \qquad (27)$$

when the loop gain equals unity and equation 26 prevails. It is interesting to observe that the frequency of oscillation is independent of the value of C_2, which can be used to set to unity the loop gain for a given value of g, a given amplitude of oscillation.

As an exercise, we will let readers verify what happens when a positive rather than a negative phase shift is introduced by the amplifier (refer to the box for a rapid conclusion).

The case of an oscillator with a non ideal amplifier

Now that we have acquainted ourselves with the subject of loop gain through two simplified cases, we are ready to study a more realistic but complex situation. As a final example, let us consider, as shown in Figure 10, a Colpitts oscillator built around a non-ideal active device. We choose to model this oscillator here by a pi model. As we will soon see, each element of this model can be derived from the *y*-parameters of the amplifier. Assuming that the reactive part of each *y*-parameter varies linearly with the frequency, in the vicinity of the oscillation frequency, each element of the pi model is frequency independent.

Evaluating the *y*-parameters of each one of the four port networks, one gets:

$$y_{11} = \left(\frac{1}{R_{in}} + \frac{1}{R_f}\right) + S\left(C_{in} + C_f\right) \qquad y_{11} = S\left(C_1 + C_2\right)$$
$$y_{12} = -\left(\frac{1}{R_f} + SC_f\right) \qquad y_{12} = -SC_1$$
$$y_{21} = -\left(g + \frac{1}{R_f}\right) + S\left(C_g - C_f\right) \qquad y_{21} = -SC_1$$
$$y_{22} = \left(\frac{1}{R_{out}} + \frac{1}{R_f}\right) + S\left(S_{out} + C_f\right) \qquad y_{22} = SC_1 + \frac{1}{SL} + \frac{1}{R_L} \tag{28}$$

Reporting those values of *y*-parameters in equation 7, one gets the following exact loop gain results :

$$\frac{\left[1 + SR_f\left(C_f + C_1\right)\right]}{\left[\frac{R_{in} + R_f}{R_{in}} + SR_f\left(C_{in} + C_f + C_1 + C_2\right)\right]}\left(g + \frac{1}{R_f}\right) \times \frac{\left[1 + S\left(\frac{C_1 + C_f - C_g}{g + \frac{1}{R_f}}\right)\right]}{\left[S\left(C_1 + C_{out} + C_f\right) + \frac{1}{SL} + \left(\frac{1}{R_L} + \frac{1}{R_{out}} + \frac{1}{R_f}\right)\right]} \tag{29}$$

One must observe that we have divided the expression of the loop gain in three parts in order to interpret the result more easily.

The first part is a first order lag-lead type transfer function of the form:

$$\left[\frac{1 + \frac{S}{\omega_1}}{A + \frac{S}{\omega_2}}\right] \tag{30}$$

where the frequencies ω_1 and ω_2 are given by:

$$\omega_1 = \frac{1}{R_f\left(C_f + C_1\right)} \quad ; \quad \omega_1 = \frac{1}{R_f\left(C_{in} + C_f + C_1 + C_2\right)} \tag{31}$$

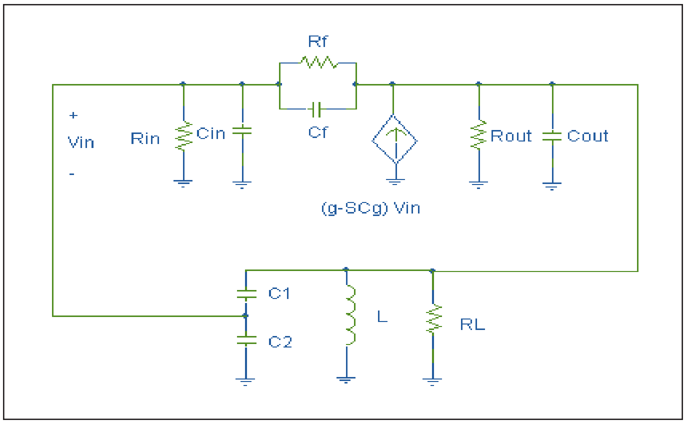


Figure 10. Variational model of a Colpitts Oscillator with a non ideal amplifier.

As $\omega_1 > \omega_2$, this transfer function introduces a local negative phase shift of up to -90° C in the frequency band comprising ω_1 and ω_2. If those two frequencies are at least ten times lower than the desired oscillation frequency ω_0, the phase shift introduced by that transfer function will be negligible at ω_0, and the first part will reduce to the following capacitive voltage divider :

$$\frac{C_f + C_1}{C_{in} + C_f + C_1 + C_2} \tag{32}$$

If this condition prevails, equation 29 can be approximately shown as equation 33 :

$$\text{Loop gain} \approx \left(\frac{C_f + C_1}{C_{in} + C_f + C_1 + C_2}\right)\left(g + \frac{1}{R_f}\right) \times \frac{\left[1 + \frac{S}{\omega_2}\right]}{\left[SC_r + \frac{1}{SL} + \frac{1}{R_r}\right]} \tag{33}$$

with

$$\omega_2 = \frac{g + \frac{1}{R_f}}{C_1 + C_f - C_g}$$
$$C_T = C_1 + C_{out} + C_f \tag{34}$$
$$\frac{1}{R_T} = \frac{1}{R_L} + \frac{1}{R_{out}} + \frac{1}{R_f}$$

The loop gain becomes purely real at a frequency given by (see Appendix):

$$\omega_0^2 = \frac{1}{L\left[C_T - \frac{1}{R_T\omega_z}\right]} \quad (35)$$

The loop gain is equal to unity when (see Appendix)

$$g + \frac{1}{R_f} = \left(\frac{C_{in} + C_f + C_1 + C_2}{C_f + C_1}\right) \times \left(\frac{1}{R_L} + \frac{1}{R_{out}} + \frac{1}{R_f}\right) \quad (36)$$

Referring to the second part of equation 33, its value is approximately equal to g because the feedback conductance ,$1/R_f$, of the amplifier is generally smaller than the forward transconductance g.

If $C_f < C_g$, it is possible to choose C_1 such that ω_z is five to ten times the value of the oscillation frequency. Doing so, one must assure that the operating point will be on the maximum phase slope portion of the bandpass at a frequency given by :

$$\omega_0^2 = \frac{1}{LC_T} \quad (37)$$

If $C_f < C_g$, or C_g is negative (which comes from a positive rather than a negative phase shift introduced by the amplifier), it may be impossible to choose C_1 so that ω_z is five to ten times the value of the oscillation frequency. In such a case, the operating point will not be located on the maximum phase slope portion of the transfer function. ■

Conclusion

The performance of an RF oscillator, regarding the starting conditions and the phase slope value at the operating point, can be determined analytically from the loop gain expression. That expression is easy to obtain if one considers the oscillator circuit as a parallel connection of two two-port networks. Furthermore, by properly regrouping the terms of the loop gain expression, one can evaluate all pertinent parameters almost by inspection. But watch out, don't confuse the loop gain with the open loop gain!

Author information

Noël Boutin is an engineer and a professor at l'Université de Sherbrooke, Faculté de Génie, Département de Génie électrique et de Génie informatique, 2500 Boul. de l'Université, Sherbrooke, Québec, Canada, J1K 2R1. His main interest is high frequency circuit analysis and design. He may be contacted by e-mail at: noel.boutin@gel.usherb.ca.

Appendix: Useful mathematical relations

IF:

$$\text{Loop gain} = \left(\frac{N}{D}\right) \cdot G \cdot \frac{\left[1 + \frac{S}{\omega_z}\right]}{\left[SC + \frac{1}{SL} + \frac{1}{R}\right]}$$

with all real variables except S,

THEN:

Characteristic equation = (1–Loop gain) = 0 =

$$S^2\left[C - \left(\frac{N}{D}\right)\cdot\frac{G}{\omega_z}\right] + S\left[\frac{1}{R} - \left(\frac{N}{D}\right)\cdot G\right] + \frac{1}{L} = 0$$

Poles are at

$$\frac{-\left[\frac{1}{R} - \left(\frac{N}{D}\right)\cdot G\right] \pm j\sqrt{4\left[C - \left(\frac{N}{D}\right)\cdot\frac{G}{\omega_z}\right]\cdot\frac{1}{L} - \left[\frac{1}{R} - \left(\frac{N}{D}\right)\cdot G\right]^2}}{2\left[C - \left(\frac{N}{D}\right)\cdot\frac{G}{\omega_z}\right]}$$

when the loop gain=1, that is, when

$$G = \left(\frac{D}{R}\right)\cdot\frac{1}{R}$$

the pole are located on the $j\omega$ axis at

$$\pm j\sqrt{\frac{1}{L\left[C - \left(\frac{N}{D}\right)\cdot\frac{G}{\omega_z}\right]}} \Rightarrow \omega_0^2 = \frac{1}{L\left[C - \left(\frac{N}{D}\right)\cdot\frac{G}{\omega_z}\right]}$$

$$= \frac{1}{L\left[C - \frac{1}{R\omega_z}\right]}$$

The phase slope of the unity loop gain at ω_0 is

$$\omega_0 = \frac{-\left[\frac{R}{\omega_0^2 L} + RC\right] + \frac{1}{\omega_z}}{\left[1 + \left(\frac{\omega_0}{\omega z}\right)^2\right]}$$