A Simple Analytic Method for Transistor Oscillator Design

This straightforward mathematical technique helps optimize oscillator designs

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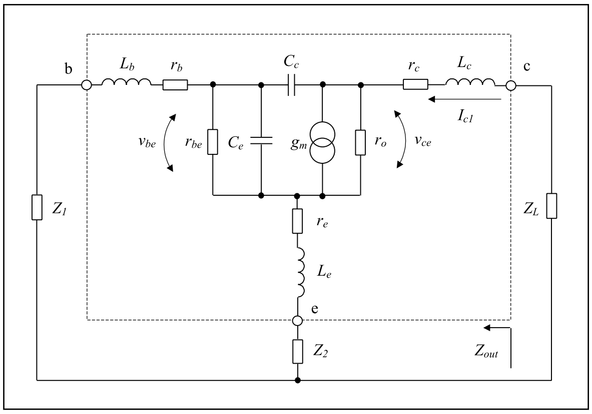
A simple analytic method for transistor oscillator design has been developed. This technique defines explicit expressions for optimum values of feedback elements and load through bipolar transistor *z*-parameters. Such an approach is useful for practical optimization of a series feedback microwave bipolar oscillator.

Microwave oscillator design in general represents a complex problem. Depending on the technical requirements for designing an oscillator, it is necessary to define the configuration of the oscillation scheme and a transistor type, to measure the small-signal and large-signal parameters of a transistor-equivalent circuit and to calculate electrical and spectral characteristics of the oscillator. This approach is very suitable for implementing CAD tools if a transistor used in microwave oscillator circuits is represented by a two-port network. There are two ways to evaluate the basic parameters of the transistor equivalent circuit; one is by direct measurement and the other is by approximating based on experimental data with reasonable accuracy in a wide frequency range [1-3]. Furthermore, the equivalent circuit model can easily be integrated into a RF circuit simulator.

In large-signal operation, it is necessary to define the appropriate parameters of the active two-port network and the parameters of external feedback elements of the oscillator circuit. Therefore, it is desirable to have an analytic method to design a single-frequency optimal microwave oscillator. This helps to formulate the explicit expressions for feedback elements, load impedance and maximum output power in terms of transistor-equivalent circuit elements and their current-voltage characteristics [4]. Such an approach can be derived based on a two-step procedure. First, the optimal combination of feedback elements for realizing a maximum small-signal negative resistance to permit oscillations at the largest amplitude is defined. Second, for a given oscillator circuit configuration with maximal output power, by taking into account the large-signal nonlinearity of the transistor equivalent circuit elements, the realized small-signal negative resistance will be characterized to determine the optimum load.



▲ **Figure 1. The series feedback bipolar oscillator equivalent circuit.**

Recent progress in silicon bipolar transistors has significantly improved frequency and power characteristics. In contrast to the field-effect transistors (FETs), the advantages of reduced low-frequency noise and higher transconductance make bipolar transistors more appealing for oscillator design up to 20 GHz. A simple analytic approach used to design a microwave bipolar oscillator with optimized feedback and load will speed up the calculations of the values of feedback elements and simplify the design procedure.

General approach

Generally, in a steady-state large-signal operation, the design of the microwave bipolar oscillator is achieved by defining the optimum bias conditions and the values of feedback elements as well as the load that corresponds to the maximum power at a given frequency. Now, lets look into the generalized two-port circuit of the transistor oscillators as shown in Figure 1, where $Z_i = R_i + jX_i$, $i = 1, 2$, $Z_L = R_L + jX_L$. Such an equivalent oscillation circuit is used mainly by microwave and radio frequency oscillator design. The dotted-lined box (as shown in Figure 1) represents the small-signal SPICE2 Ebers-Moll model of the bipolar transistor in the normal region of operation. This hybrid-p model can accurately simulate both DC and high-frequency behavior up to the transition frequency $f_T = g_m/2\pi C_e$ [5]. For generic microwave bipolar oscillator design, the oscillation will arise under capacitive reactance in an emitter circuit ($X_2 <0$), inductive reactance in a base circuit ($X_1 >0$), and either inductive ($X_L >0$) or capacitive ($X_L<0$) reactances in a collector circuit.

For a single frequency of oscillation, the steady-state oscillation condition can be expressed as

$$Z_{out}(I,\omega)+Z_L(\omega)=0 \tag{1}$$

where $Z_L(\omega)=R_L(\omega)+jX_L(\omega)$ and $Z_{out}(I, \omega)= R_{out}(I, \omega)+ jX_{out}(I, \omega)$.

The expression of output impedance, $Z_{out,}$ can be written as

$$Z_{out} = Z_{22} + Z_2 - \frac{(Z_{12}+Z_2)(Z_{21}+Z_2)}{Z_{11}+Z_2+Z_1} \tag{2}$$

where Z_{ij} ($i, j = 1, 2$) are z-parameters of the hybrid transistor model.

To optimize the oscillator circuit, the negative real part of the output impedance Z_{out} has to be maximized. Based on expression (2), it is possible to find optimal values for X_1 and X_2 under which the negative value R_{out} is maximized by setting

$$\frac{\partial R_{out}}{\partial X_1}=0, \quad \frac{\partial R_{out}}{\partial X_2}=0 \tag{3}$$

The optimal values X_1^0 and X_2^0 based on condition (3) can be expressed with the impedance parameters of the active two-port network in the following manner [4]:

$$X_1^0 = \frac{R_{21}-R_{12}}{X_{21}-X_{12}}\left(\frac{R_{12}+R_{21}}{2} - R_{11} - R_1\right) - X_{11} + \frac{X_{12}+X_{21}}{2}$$

$$X_2^0 = -\frac{(2R_2+R_{12}+R_{21})}{2(X_{21}-X_{12})} - \frac{X_{12}+X_{21}}{2} \tag{4}$$

By substituting X_1^0 and X_2^0 into equation (2), the optimal real and imaginary parts of the output impedance Z_{out} can be defined as follows:

$$R_{out}^0 = R_2 + R_{22} - \frac{(2R_2+R_{12}+R_{21})^2 + (X_{21}-X_{12})^2}{4(R_{11}+R_2+R_1)}$$

$$X_{out}^0 = X_2^0 + X_{22} - \frac{R_{21}-R_{12}}{X_{21}-X_{12}}\left(R_{out}^o - R_2 - R_{22}\right) \tag{5}$$

Small-signal oscillator circuit design

At radio and microwave frequencies, the condition $r_{be} >>1/\omega\, C_e$ is usually fulfilled. Besides, it is possible to ignore the effect of base-width modulation (the so-called Early effect) without a significant decrease of the final result accuracy, and to consider the resistance r_o as an infinite value. The parasitic lead inductances and substrate capacitance can be taken into account in the external feedback circuit. By doing so, the internal bipolar transistor in common-emitter small-signal operation can be characterized by the following real and imaginary parts of z-parameters:

$$R_{11}=R_{12} = a \times \left[\frac{1}{g_m} + r_b\left(\frac{\omega}{\omega_T}\right)^2\right]$$

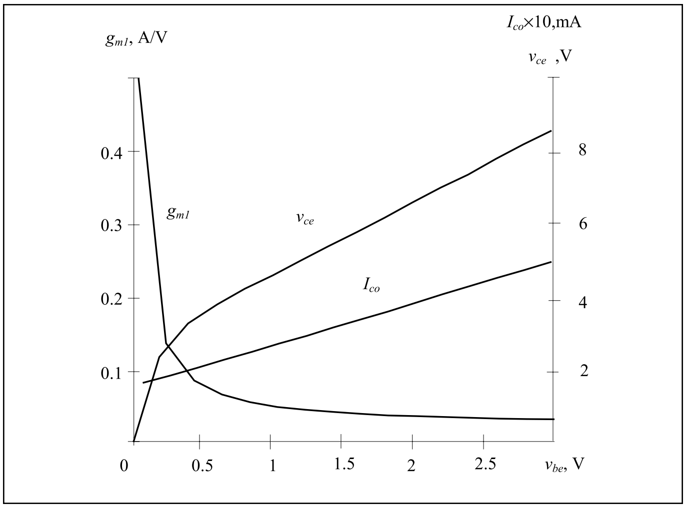
$$R_{21} = R_{22} = R_{11} + \frac{a}{\omega_T C_c}$$

$$X_{11} = X_{12} = -a\frac{\omega}{\omega_T}\left(\frac{1}{g_m} - r_b\right) \tag{6}$$

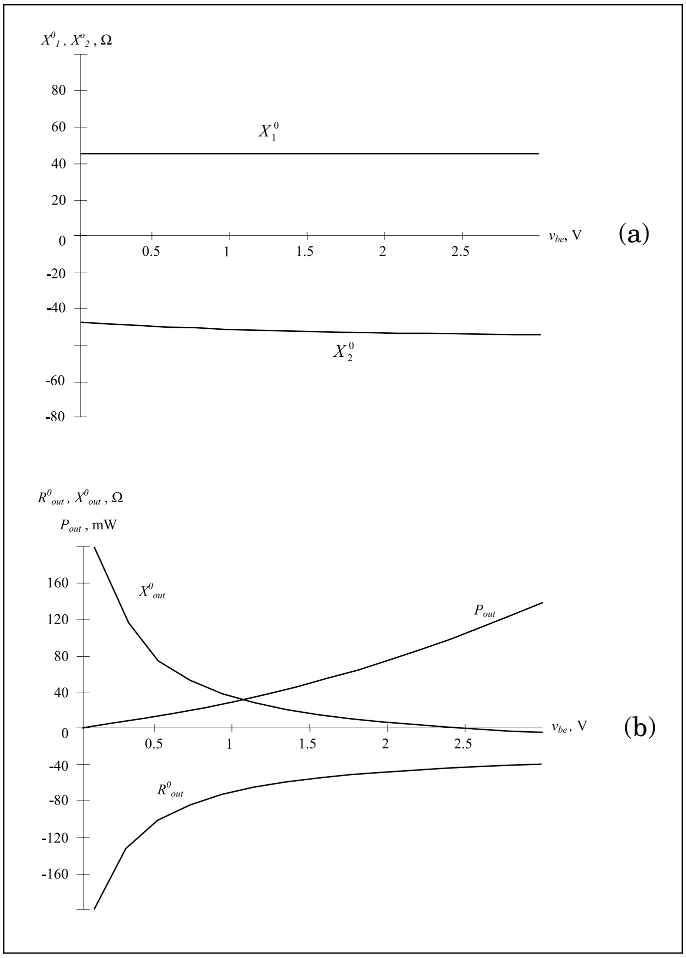
$$X_{21} = X_{11} + \frac{a}{\omega C_c}\left(\frac{\omega}{\omega_T}\right)^2$$

$$\text{where } a = 1\Big/\left[1+\left(\frac{\omega}{\omega_T}\right)^2\right] \omega_T = 2\pi f_T$$

By substituting the expressions for real and imaginary parts of the transistor z-parameters from the system of equations (6) to equations (4) and (5), the optimal



▲ **Figure 2. Amplitude dependencies of large-signal transconductance, constant bias collector current and fundamental amplitude.**



▲ **Figure 3. Amplitude dependencies of (a) optimum cicuit parameters and (b) the real and imaginary parts of output resistance and output power of the bipolar oscillator.**

values of imaginary parts of the feedback elements X_1^0 and X_2^0 can be rewritten as follows:

$$X_1^0 = \frac{1}{2\omega C_c} - r_b \frac{\omega}{\omega_T}$$
$$X_2^0 = -\frac{1}{2\omega C_c} - r_e \frac{\omega}{\omega_T} \tag{7}$$

In addition, the real and imaginary parts of optimum output impedance

$$Z_{out}^0 = R_{out}^0 + jX_{out}^0$$

can be expressed as:

$$R_{out}^0 = r_c + \frac{r_b}{r_b + r_e + R_{11}}\left(r_e + R_{11} + \frac{a}{\omega_T C_e}\right) - \frac{a}{r_b + r_e + R_{11}}\left(\frac{1}{2\omega C_e}\right) \tag{8}$$

$$X_{out}^0 = \frac{1}{2\omega C_c} - \left(R_{out}^0 - r_c\right)\frac{\omega}{\omega_T}$$

From equation (8), it follows that as frequency increases, the absolute value of the negative resistance R^0_{out} will be reduced and the maximum oscillation frequency f_{max} becomes zero. Without considering the parasitic series resistors r_e and r_c, the expression for f_{max} is

$$f_{\max} = \sqrt{\frac{f_T}{8\pi r_b C_c}} \tag{9}$$

Equation (9) matches with the well-known expression for f_{max} of the bipolar transistor, on which a maximum power amplification factor is unity, and the steady-state oscillation condition is carried out solely for the lossless oscillation system [6].

Large-signal oscillator circuit design

Generally, at least three steps are required for designing a large-signal oscillator circuit. First, it is necessary to choose an appropriate circuit topology, second, to determine the device large-signal characteristics and, third, to optimize the oscillator circuit to achieve the desired performances. To analyze and design the fundamental negative resistance bipolar oscillator, it is advisable to apply a quasi-linear method, based on the use of the ratios between the fundamental harmonics of currents and voltages as well as the representation of nonlinear elements by equivalent averaged fundamental linear ones. The derivation of equivalent linear elements in terms of signal voltages is based on static and voltage-capacitance bipolar transistor characteristics. In a common case, all elements of the transistor equivalent circuit are nonlinear and depend significantly on operation mode, especially for transconductance g_m and base-emitter capacitance C_e. However, in practice, it is enough to be limited to the nonlinear elements g_m, ω_T and C_c, as the base resistance r_b poorly depends on a bias condition. Besides, to calculate the constant bias and fundamental collector currents, it is sufficient to use a linear approximation of transition frequency ω_T and the small-signal value C_c at operating point [4]. Under the assumption of a sufficiently low collector-emitter bias value, the applied bipolar transistor model does not represent the effect of the forward-rectified current across the collector-base junction and the collector voltage-breakdown phenomenon. In this case, it is assumed that the transition from a soft-excited oscillation mode to a steady-state stationary mode is caused mainly by the nonlinear characteristic of the transconductance g_m in contrast to the FET oscillator. Here, under large-signal operation both the oscillator impedance and power prediction are primary due to a change in differential drain-source resistance [4].

When the base-emitter current is sufficiently small, the following exponential model can be applied to approximate the family of the experimental transfer I-V characteristics:

$$I_c = I_{cs}\left[\exp(V_{be} / V_T) - 1\right] \tag{10}$$

where V_{be} is the base-emitter junction voltage, I_{cs} is the reverse collector saturation current, and V_T is the temperature voltage. By restricting to the fundamental frequency, V_{be} can be expressed as

$$V_{be} = E_b - E_e - I_{co}R_e + v_{be}\cos\omega\, t,$$

where I_{co} is a constant bias collector current, E_b and E_e are the base and emitter bias voltages, and R_e is a self-bias resistor.

Taking into account that the voltage drops on collector and emitter transistor equivalent circuit resistors r_c and r_e are negligible, the values of the large-signal transconductance g_{m1} and the constant bias collector current I_{co} as the functions of the junction fundamental voltage amplitude v_{be} can then be defined as follows:

$$g_{m1} = \frac{21_{cs}}{v_{be}} \exp\left(\frac{E_b - E_e - I_{co}R_e}{V_T}\right) \cdot I_1\left(\frac{v_{be}}{V_T}\right) \tag{11a}$$

$$I_{co} = I_{cs}\left[I_0\left(\frac{v_{be}}{V_T}\right) \exp\left(\frac{E_b - E_e - I_{co}R_e}{V_T}\right) - 1\right] \tag{11b}$$

where $I_0(v_{be} / V_T)$, $I_1(v_{be} / V_T)$ are the modified first order Bessel functions [7].

For the steady-state stationary oscillation mode, it is necessary to define the analytic relations between the load current amplitude and fundamental amplitude of collector voltage with input voltage amplitude v_{be}. These relations are expressed in terms of the transistor z-parameters and oscillator circuit parameters in the following manner:

$$I_{cl} = \frac{Z_{11} + Z_2 + Z_1}{Z_{11}Z_2 - Z_{12}(Z_2 + Z_1)} V_{be}$$

$$V_{ce} = \frac{Z_{22}(Z_{11} + Z_2 + Z_1) - Z_{21}(Z_2 + Z_{12})}{Z_{12}(Z_1 + Z_2) - Z_{11}Z_2} \tag{12}$$

In a steady-state operation, the output power of the oscillator is

$$P_{out} = I_{cl}^2 \operatorname{Re} Z_L / 2$$

By replacing load resistance Z_L from equation (1) and using the parameters of transistor equivalent circuit from (6) to (8), P_{out} can be expressed as

$$P_{out} = -a g_{ml}^2 (r_b + r_e + R_{11}) \frac{R_{out}^0}{r_b + r_c - R_{out}^0} \cdot \frac{v_{be}^2}{2} \tag{13}$$

Results

An analytic approach was applied to the microwave bipolar oscillator design. The bipolar transistor has the following parameters of its equivalent circuit: f_T = 6 GHz, C_c = 0.5 pF, g_m = 1.6 A/V, r_b = 4 ohms, r_e = 0.3 ohm, r_c = 1.75 ohms, $L_e = L_c$ = 0.5 nH, L_b = 0.3 nH. The numerical calculation was performed using the values of the oscillation frequency f = 4 GHz, biases E_b = 0 V and E_e = –2 V, self-bias resistor R_e = 100 ohms and reverse saturation collector current I_{cs} = 10 µA. The numerical results obtained are shown in Figures 2 and 3. Figure 2 shows the amplitude dependence of large-signal transconductance g_{m1}, constant bias collector current I_{co}, and fundamental collector amplitude v_{ce}. Figure 3 shows the amplitude dependence of (a) optimum circuit parameters, X_1^0 and X_2^0 (b) the real and imaginary parts of the output resistance R_{out}^0, X_{out}^0 and output power P_{out}.

According to Kurokawa [8], as the negative resistance R_{out}^0 reduces with the increase of the base-emitter junction and collector voltage amplitudes, the stable oscillations are established in the oscillator. In that case, the value of the large-signal transconductance g_{m1} is significantly reduced as shown in Figure 2. In order to realize the maximal or required output power in a linear operation region without the saturation effect, it is necessary to choose the appropriate load value or to use the diode in parallel to load for restriction of collector voltage amplitude. The applied model of the microwave bipolar transistor does not include a collector-base diode, as such an operation region is not recommended for the oscillator due to the significant deterioration of its noise properties. By means of the condition $v_{ce} \le E_c$, the appropriate amplitude restriction at the numerical calculation can be considered.

From numerical calculations, it follows that v_{ce} and I_{co} are increased linearly with the increase of base-emitter junction amplitude v_{be}, starting from some significantly small values as shown in Figure 2. Therefore, analytically it is interesting to consider the influence of the emitter self-bias resistor R_e on the character of amplitude dependence of the fundamental output current I_{c1}. The fundamental output current can be expressed as

$$I_{cl} = 21_{cs}\left[\exp\left(\frac{E_b - E_e - E_{co}R_e}{V_T}\right) \times I_1\left(\frac{v_{be}}{V_T}\right)\right] \tag{14}$$

By differentiating, equation (11b) can be rewritten based on $d_{I0}(v_{be}/V_T)/d(v_{be}/V_T) = I_1(v_{be}/V_T)$ in the form of

$$\frac{dI_{co}}{dv_{be}} = \frac{I_{co} + I_{cs}}{V_T + (I_{co} + I_{cs})R_e} \cdot \frac{I_1(v_{be}/V_T)}{I_0(v_{be}/V_T)} \tag{15}$$

From the definition of modified first kind Bessel functions, it follows that for values $(v_{be}/V_T) \le 5$ the following inequality is to be defined:

$$0.9 \le I_1(v_{be}/V_T) / I_0(v_{be}/V_T) \le 1$$

Hence, as it follows from (15), the value of constant bias collector current I_{co} varies practically linearly with the increase v_{be} under condition $v_{be} \ge 5V_T$, and the slope of the dependence $I_{co}(v_{be})$ is defined by value R_e. By substituting expression (11b) into equation (14), it allows us to rewrite the expression for fundamental output current I_{c1} as

$$I_{cl}(v_{be}) = 2\left[I_{co}(v_{be}) + I_{cs}\right] \times \frac{I_1(v_{be}/V_T)}{I_0(v_{be}/V_T)} \tag{16}$$

From equations (15) and (16) it follows that for a bipolar oscillator with an emitter self-bias resistor, the collector constant bias current I_{co} and the fundamental output current I_{c1} depend linearly on the base-emitter junction voltage amplitude v_{be} under the condition $v_{be}/V_T \ge 5$. Thus, the linearizing influence of the emitter self-bias resistor for the transistor oscillator is similar to the influence of the negative feedback resistor on a linearization of the amplifier gain-transfer characteristics. From Figure 3b, it follows that it is possible to realize a sufficiently high level of output power under the direct connection of standard load R_L = 50 ohms without use of a special load matching circuit.

Conclusion

A simple analytic method of microwave and RF bipolar oscillator design has been developed, allowing the definition of explicit expressions for optimum values of feedback elements and load through bipolar transistor z-parameters. A negative resistance concept is used to design series feedback microwave and RF bipolar oscillators with optimized feedback elements and maximum output power in terms of transistor impedance parameters. Based on its small-signal *z*-parameters and DC characteristics, a simplified large-signal model for a bipolar transistor is derived. The linearizing effect of the external emitter self-bias resistor is analytically shown. ■

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