# Understanding the Sources of Phase Measurement Errors

Precision measurements are affected by temperature, mismatches, calibration standards and the accuracy of instruments and cables

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Phase measurements are subject to many errors. The primary sources of error are temperature variations during measurement, VSWR mismatch between the test port and the device, and test equipment accuracy. The equipment accuracy is a combination of errors in the instrument, the calibration standards, and the test leads.

The first source of error, temperature variations during measurement, is easily controlled as the calibration and measurement should take only a few minutes. For measurements taking longer, the equipment must be maintained at a constant temperature. It is not always practical to maintain this type of control over test leads. Teflon® test leads are one option, but unfortunately, because of the molecular structure of PTFE the phase response is not repeatable around room temperature. $SiO_2$ test leads on the other hand have a very repeatable phase response over a very wide temperature range. To reduce error due to temperature, keep in mind that small changes in temperature affect dielectric constant while larger changes affect both dielectric constant, and physical length. Both factors affect the measured phase.

The second source of error, VSWR mismatch between the test port and the device, can be calculated. One commonly used formula can be found in the Hewlett Packard "Mismatch Error Limits Calculator."

$$\text{Max Phase Error} = \pm \sin^{-1}\left|\frac{SWR_1 - 1}{SWR_1 + 1} \cdot \frac{SWR_2 - 1}{SWR_2 + 1}\right|$$

For VSWR mismatches below 1.1:1, the phase error is less than 0.15 degrees. However for VSWR mismatches of 1.5:1 the error can be over 2 degrees. VSWR error is not generally significant as network analyzers are calibrated before use. Results will vary depending on which calibration is used. The "thru" calibration is faster, but more accurate results will be obtained with the full two port calibration.

The third source of error, test equipment accuracy, is documented in the manufacturer's specifications. Although documented, the user may wish to quantify the actual performance of the specific equipment being used, or at least verify the equipment is operating correctly.

## Estimation of phase accuracy in a vector network analyzer

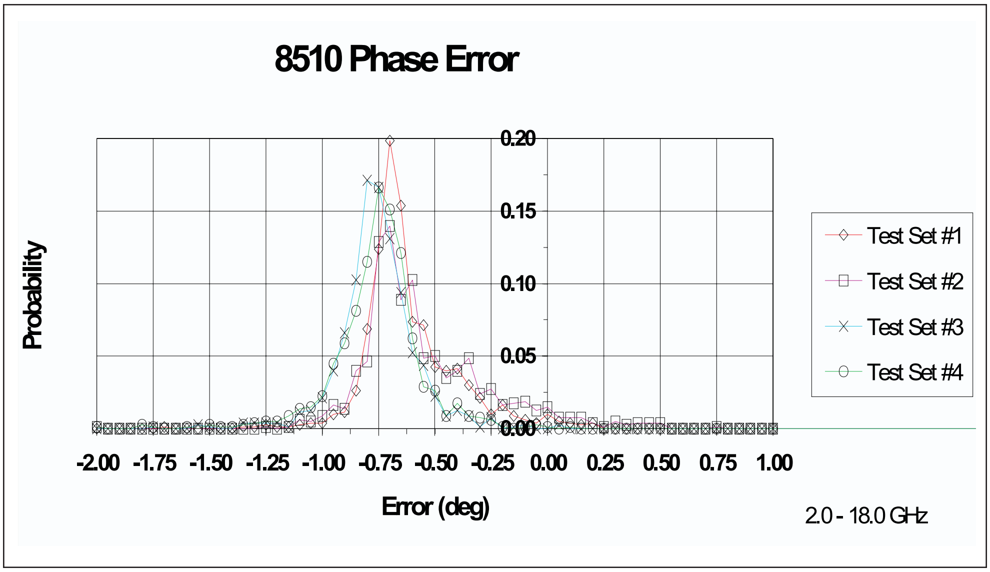
Vector network analyzers are state of the art instruments for the measurement of phase. Their specified errors are only a few degrees, even at 18 GHz. The following procedure may be used to develop a rough estimation of the insertion phase accuracy of a network analyzer.

The analysis is performed using an insertable 3.5 mm precision airline. The airline is used as the insertion phase response is linear over frequency. Justifaction for this assumption is established later in this article.

An airline has very low input VSWR, less than 1.05:1 for DC - 18.0 GHz. Since it is gold plated with no inner dielectric, the phase response is very predictable and linear over a wide frequency range.

To conduct the analysis, the network analyzer is calibrated with a high number of samples (801 or more), using 3.5 mm connectors configurated for an insertable device. Insertion phase data is taken for the airline, and reduced to absolute phase to remove the 180° rollovers. The measured phase is subtracted from the theoretical. What remains is the phase error for each frequency. Small effects of discontinuities are ignored. This is justified by considering that the test port to airline discontinuities are similar to those of the test port to calibration standards. Further justification is based on the low measured VSWR of the airline.

Ideally, the phase error will be zero, but analysis of the data shows that the error varies over frequency, as expected. This error is applied to any length of cable with a loss roughly that of the airline. To quantify errors for a cable with greater loss, the network analyzer attenuators are set to provide loss equal to that of the cable under consideration, and the analysis is repeated. To justify this concept, consider the following factors. First, the detectors of the network analyzer measure the magnitude and phase of the $s_{21}$ parameter. Next, the loss of the airline is fairly constant over frequency (±1 dB from 2.0 to 18.0 GHz) and its insertion phase rotates repeatedly through 180° to –180° as the frequency increases. Since the power level is fairly constant, the error measured is the error of the detectors for various phase angles. This is equivalent to varying the length of the cable. Caution should be used with this approach, as the analyzer does not use the same detectors for all frequencies.




▲ **Figure 1. Estimated probability for error in four HP 8510 network analyzers.**

## Results

Figure 1 shows the estimated probability of error for phase measurements in the HP 8510C network analyzers at Kaman Instrumentation. Hewlett Packard performed a system analysis of these analyzers and predicts a worst case error of:

2.0 - 8.0 GHz: ±1.1°
8.0 - 18.0 GHz: ±2.9°

Our measurements indicate the following phase performance for our analyzers:

Test set #1 has a phase error of –0.68°, +0.8°/–0.4°

Test set #2 has a phase error of –0.68°, +1.2°/–0.5°

Test set #3 has a phase error of –0.81°, +0.7°/ –0.5°

Test set #4 has a phase error of –0.75°, +0.75°/–0.5°

## Repeatability

Measurement repeatabilty is very good. Repeatability specifications for the HP8510s at Kaman Instrumentation are less than 0.25 degrees over 2 to 18 GHz. Parameters affecting repeatability are use of consistent torque values, test leads configured similarly, consistent room temperature, and clean connectors.

## Comments

This article describes the various sources of phase error in measurements made with a vector network analyzer. The procedure discussed for quantifying the phase error in a network analyzer is only a rough approximation. A more precise estimate is obtainable by performing a verification test. Such tests are performed using a supplier's verification kit. These verification tests can provide accuracies traceable to NIST.

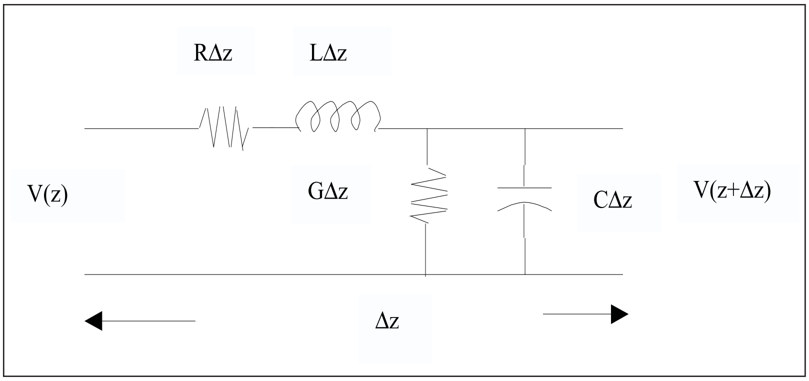
## Phase constant of a coax cable

The following paragraphs present the derivation of the phase coefficient of coax cable. The derivation shows the phase constant of an airline can be assumed linear over frequency. This is also supported by statistical analysis of the data taken (not shown).

Coaxial cable is a dual conductor transmission line. Typically, all transmission lines are modeled with a series resistance, series inductance, parallel capacitance and parallel conductance. The parameters are based on a differential length. One such model is shown in Figure 2.

It can be shown that the voltage anywhere on the cable satisfies the following equation.

$$V(z) = V^{+}e^{-\alpha z}e^{-j\beta z}e^{j\omega t} + V^{-}e^{\alpha z}e^{j\beta z}e^{j\omega t}$$




▲ **Figure 2. Coax cable distributed element model.**

where $V^+$ is the amplitude of the forward traveling wave and $V^-$ is the amplitude of the reverse traveling wave.

The attenuation and phase constants are:

$$\alpha = k\sqrt{\frac{-1+\sqrt{1+\rho^2}}{2}}$$

$$\beta = k\sqrt{\frac{1+\sqrt{1+\rho^2}}{2}}$$

$$k = \sqrt{\omega^2 LC - RG}$$

$$\rho = \frac{\omega^2(LG - RC)}{\omega^2 LC - RG}$$

$$\omega = 2\pi f$$

For a coax cable of dimensions (a,b) the distributed elements R, L, C, & G are calculated as follows:

$$L = \frac{\mu_d \mu_0}{2\pi}\ln\frac{b}{a} \quad H/m$$

$$C = \frac{2\pi\varepsilon_d\varepsilon_0}{\ln\left(\frac{b}{a}\right)} \quad F/m$$

$$R = \frac{1}{2\pi}\sqrt{\frac{\omega\mu_c\mu_0}{2\sigma_c}}\left(\frac{1}{a}+\frac{1}{b}\right) \quad \Omega/m$$

$$G = \frac{2\pi}{\ln\left(\frac{b}{a}\right)}\left(\omega\varepsilon_d\varepsilon_0 \tan\delta - \sigma_d\right) \quad S/m$$

where,

$\mu_0 = 4\pi\times10^{-7}$
$\mu_{c/d}$ = relative permeability of the conductor/ dielectric

$\varepsilon_0 = 8.854\times10^{-9}$
$\varepsilon_d$ = realtive permittivity of the dielectric

$\sigma_c$ = conductivity of the conductor
$\sigma_d$ = conductivity of the dielectric

$\tan\delta$ = loss tangent of the dielectric

For a precision 3.5 mm gold plated airline the phase constant reduces to

$f = 2.0 - 18.0\ \text{GHz}$

$\varepsilon_d = 1$

$\mu_{c/d} = 1$

$C = 7.535\varepsilon_0$

$L = 0.133\mu_0$

$G = 0$

$R = 20\times10^{-6}\sqrt{f}$

$\rho = \frac{19}{\sqrt{f}} \approx 0$

$k = 2\pi f\sqrt{\mu_0\varepsilon_0}$

$\beta = 2\pi f\sqrt{\mu_0\varepsilon_0}$

Thus the phase constant ($\beta$) is linear and smooth. ■

## Reference

1. D. M. Pozar, *Microwave Engineering*, 1990.

## Author information

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