Broadband, VHF Lumped–Element Couplers Using Imperfect Transformers

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This paper describes an approach to the design of VHF lumped-element hybrid couplers that have broad bandwidth and do not require perfect magnetic coupling between the inductors. This design may be a useful alternative in cases where nearly ideal coupling cannot be achieved.

The "classical" design for a lumped-element VHF quadrature hybrid was described 20 years ago by Ho and Furlow [1, 2]. This circuit, shown in Figure 1, uses a pair of perfectly coupled inductors (i.e., having a unity coupling coefficient) and two capacitors. The simple single-section coupler, however, is narrowband, but the bandwidth can be increased substantially by connecting two of them by transmission lines. Unfortunately, the transmission lines can be uncomfortably long, especially at low frequencies.
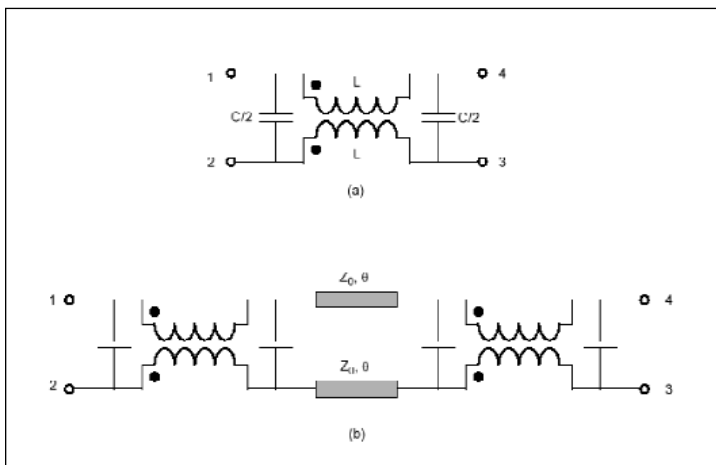
Design equations for this coupler are fairly simple. We first define the coupling ratio, k:

$$k = \left| \frac{s_{21}}{s_{41}} \right| \tag{1}$$

and the design equations are:

$$L = \frac{kZ_0}{\omega} \tag{2}$$

$$C = \frac{L}{Z_0^2}$$

▲ **Figure 1. Single (a) and multisection (b) couplers from references [1] and [2]. The inductors are assumed to have perfect magnetic coupling and the transmission-line impendance Z_0 is equal to the port impendance.**

where L and C are as shown in the figure, and ω is the center frequency.

Since the magnetic coupling must be perfect, the mutual inductance of the coupled inductors must equal the self inductance, L. In practice, tight coupling is achieved by winding the coils as a twisted pair of wires on a ferrite toroid.

A single-section coupler of this type is relatively narrowband. To increase the bandwidth, two identical sections can be connected by transmission lines, as shown in Figure 1b. Ho and Furlow give a process for determining θ, but it is a little cumbersome. A simpler method is to use a circuit simulator's tuning capability to find θ. The coupling is a little sensitive to the value of θ with slight changes in θ, resulting in an overcoupled or undercoupled response.

Furthermore, the transmission lines tend to shift the center of the coupling band lower in frequency. Thus, it is necessary to design the individual sections to have a center frequency somewhat higher than that of the finished coupler. When used as a 3 dB hybrid, the two-section coupler is capable of bandwidths of 60 percent or more.

For a 3 dB hybrid coupler, typical transmission-line lengths are 25 to 45 degrees at the coupler's center frequency. At 350 MHz, for example, this is approximately 2 to 3 inches, depending on the dielectric constant of the substrate used for the transmission lines. In many applications, especially at lower frequencies, this is uncomfortably long. Clearly, it would be useful to have a broadband coupler design that did not require the transmission lines.

An alternate design

It is possible to use an alternate design that requires no transmission lines, is broadband, and does not require perfect coupling between the inductors. As such, it may be useful in situations where toroidal inductors are impractical or where long transmission lines are intolerable. Figure 2 shows the new design.

The structure of the coupler approximates a set of coupled transmission lines. The self inductances of the inductors and the capacitors C_1 approximate the individual lines, and the mutual inductance and capacitors C_2 provide the coupling. Unfortunately, there must be both inductive and capacitive coupling, because capacitive coupling alone is insufficient to obtain good performance. When the coupler is excited at port 1, the "through" port is port 4 and the coupled port is port 2. Port 3 is isolated.

Because its L-C cutoff is too close to the center frequency of the coupler, the single-section structure, shown in Figure 2a, is probably adequate only for narrowband applications. A two-section coupler, shown in Figure 2b, is usually more satisfactory. The bandwidth can be extended slightly by using more sections, but the improvement is minor and arguably not worth the additional cost.
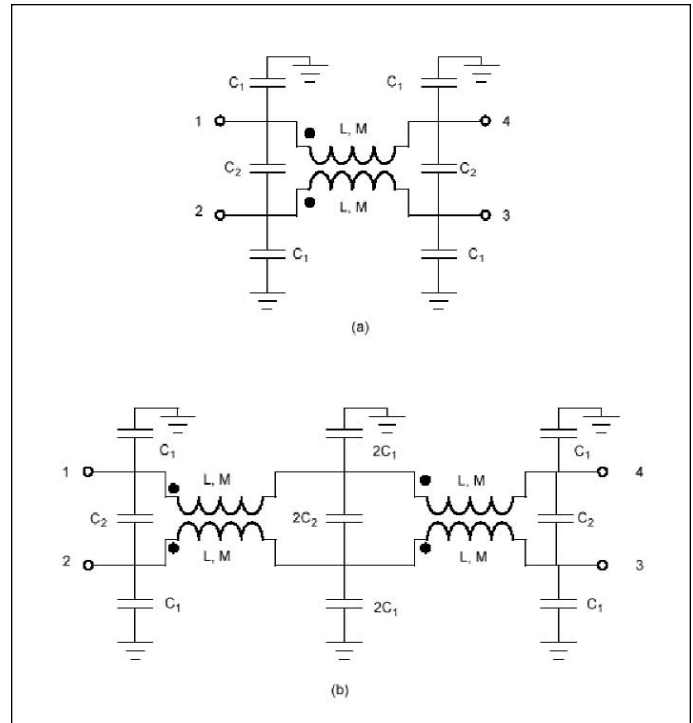
The design equations are as follows. We first define the coupling K slightly differently from [1]:

$$K = |s_{21}| \tag{3}$$

then we have

$$M = KL \tag{4}$$

$$C_1 = \frac{L - M}{2Z_0^2} \tag{5}$$

▲ Figure 2. The circuit of the single-section coupler (a) and the two-section coupler (b). The inductors do not have perfect coupling. Because the L-C cutoff is too close to the center frequency, the single-section coupler is probably not practical for most applications.

$$C_2 = \frac{MC_1}{(L - M)} = \frac{KC_1}{1 - K} \tag{6}$$

where M is the mutual inductance and C_1, C_2 and L are as shown in Figure 2.

The center frequency f_0 is:

$$f_0 = \frac{1}{4n\sqrt{2(L + M)C_1}} \tag{7}$$

where n is the number of sections in the coupler. The approximate L-C cutoff frequency of the coupler is

$$f_c = \frac{1}{\pi\sqrt{2(L + M)C_1}} \tag{8}$$

so the ratio of the cutoff frequency to the center frequency is simply:

$$\frac{f_c}{f_0} = \frac{4n}{\pi} \tag{9}$$

Equation 9 shows clearly why a single-section coupler ($n = 1$) has poor characteristics: the cutoff frequency is only 27 percent above the center frequency! Because of this characteristic, a single-section 3 dB coupler has only about 25 percent bandwidth. However, a two-section coupler has $f_c = (8/\pi)f_0 = 2.54 \, f_0$. This is well outside the upper band edge of the coupler.

Design example

Let us design a two-section, 3 dB hybrid coupler centered at 355 MHz. First, from (3) we determine the coupling $K = 0.707$, and (4) gives the ratio of M to L. We select a trial value for L, then find M and C_1 from (6). We substitute these values in (7) to determine the center frequency, then modify L and repeat the process until the center frequency is right. Finally, we calculate C_2 from (6). After a couple of iterations, we obtain

$L = 25$ nH
$M = 17.1$ nH
$C_1 = 1.47$ pF
$C_2 = 3.52$ pF

Figure 3 shows the coupling, isolation, and return loss, calculated on the Microwave Office Design Suite [3]. The coupling decreases 0.5 dB at 245 and 435 MHz, giving a center frequency of 340 MHz and bandwidth of 56 percent. Figure 4 shows the phase. The phase balance between the "through" and coupled ports is well within one degree over this band. The return loss and isolation are nearly identical over this frequency range and are greater than 23 dB.

Greater bandwidth could be achieved by slight over-coupling at the center frequency, reducing the coupling error at the band edges.

Determining *L* and *M*

It is difficult to design coupled inductors for specific values of L and M, but it is relatively easy to measure these parameters. Equations for the self inductances of the windings are well known, and those expressions can be used to design the individual windings. The distance between the windings can then be varied empirically to achieve the desired value of M.

The Z matrix of a pair of coupled inductors is

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = jw \begin{bmatrix} L & M \\ M & L \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad (10)$$

where the inductances of the two windings are identical, and the terminals of each winding realize a port. The pair of inductors is treated as a two-port and is measured with a network analyzer. The resulting s-parameters are converted to z-parameters, and the self and
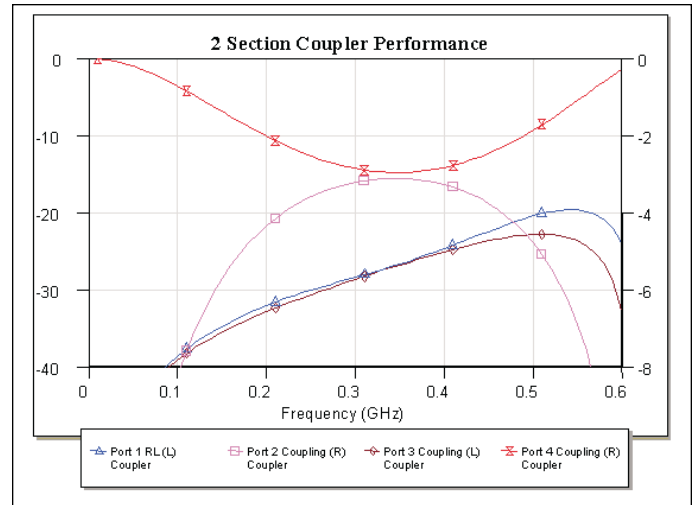
▲ **Figure 3. Coupling, return loss and isolation of the 355 MHz, 3 dB coupler.**
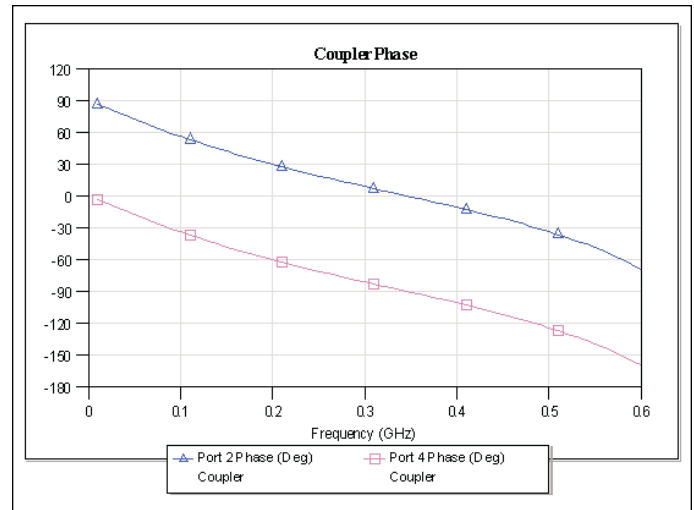
▲ **Figure 4. Phase of the 355 MHz coupler's "through" and coupled ports.**

mutual inductances are found as

$$L = \frac{Z_{11}}{j\omega} \qquad M = \frac{Z_{21}}{j\omega} \qquad (11)$$

It is best to calculate L and M from (11) over the entire frequency range of the coupler, and to average them to obtain values for use in the design. Large deviations from the average are an indication that the inductor may have too much stray capacitance. ∎

References

1. C. Y. Ho and R. Furlow, "Design of VHF Quadrature Hybrids, Part I," *RF Design*, July/August 1979.

2. C. Y. Ho and R. Furlow, "Design of VHF Quadrature Hybrids, Part II," RF *Design*, September/October 1979.

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Author information

Steve Maas is a consulting engineer specializing in low-noise and nonlinear circuits and systems. He has written a few books on these subjects and can usually be spotted lurking around technical symposia that have sessions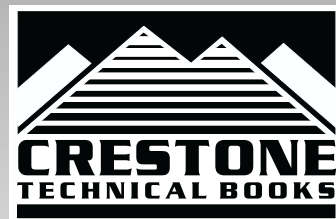 on nonlinear circuit analysis. He is the creator of the nonlinear circuit analysis engine in Microwave Office from Applied Wave Research. He has also worked in the Electrical Engineering department at UCLA, served on the MTT Adcom and has been the editor of the IEEE MTT Transactions. He can be reached at Nonlinear Technologies, Inc., P. O. Box 7284, Long Beach, CA 90807, USA. Steve may be reached via e-mail at: smaas@nonlintec.com