Wireless Digital Modulation

Digital modulation techniques commonly used in wireless communication systems — PSK, MSK and FSK — are described in this paper with respect to their power spectral density, bandwidth and probability of bit error.

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Modern wireless communication systems use digital modulation techniques to transmit data forming a modulation (modulating signal) in binary or M-ary coded form on an RF carrier which normally is a single sinusoidal wave.

The modulation techniques currently employed in modern communication systems can be classified into three major categories: Amplitude Shift Keying, Phase Shift Keying and Frequency Shift Keying as follows:

Amplitude Shift Keying (ASK) occurs when the amplitude of the transmitted signal is varied as a function of the data.

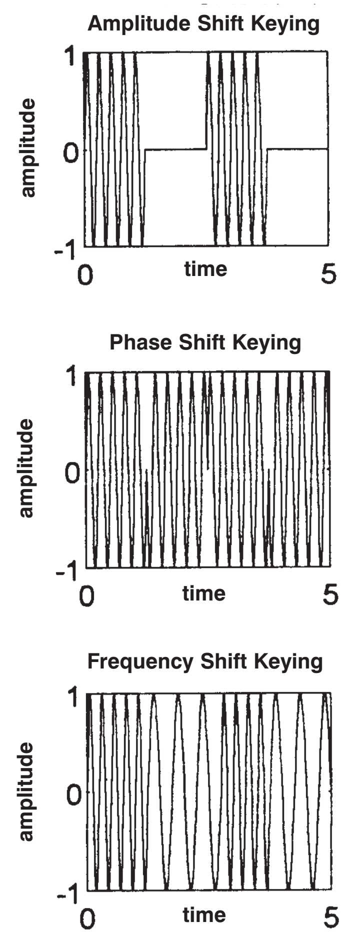
Phase Shift Keying (PSK) occurs when the phase of the transmitted signal is varied as a function of the data.

Frequency Shift Keying (FSK) occurs when the frequency of the signal is varied as a function of the data.

Figure 1 illustrates the three schemes in a time-domain representation. Since ASK requires the amplitude of the signal to contain information, it is not widely used in communication systems because of the resulting amplitude variation of the RF signal and the consequent need for linear RF amplifiers if the modulation is to be transmitted and received faithfully. For this reason we will not consider it further in this paper.

Demodulation performed at the receiver can be either coherent or non-coherent. Coherent demodulation occurs when the carrier phase information is retrieved from the transmitted signal and used to extract the data. On the other hand, non-coherent demodulation takes place when the RF carrier phase information is not employed in the demodulation.

Coherent demodulation has the advantage that the Probability of Symbol Error (P_E) at reception is better. In other words, there is a lower probability of error than will be encountered in a noncoherent scheme, assuming the same signal power is received for either trans-



■ **Figure 1. Time domain representations of ASK, PSK and FSK.**

mission. Of course, the coherent method requires extra circuitry to extract the phase information from the RF carrier and is therefore likely to be more expensive and bulky to implement. Nevertheless, coherent demodulation is more faithful and most commonly used because signal power and bandwidth always are at a premium. Accordingly, all modulation schemes considered herein are coherent.

There are several factors that influence the selection of a modulation technique including bandwidth efficiency (data rate divided by bandwidth) of the system, transmitted signal power required for detection, probability of bit error at reception and circuit complexity considerations. Many of these factors are interrelated, and an improvement in one generally means a degradation in another.

Phase Shift Keying (PSK)

In PSK, the phase of the RF carrier is changed as a function of the data being transmitted. The amplitude and frequency of the RF carrier remain constant. Mathematically, PSK is represented by:

$$PSK(t) = A \cos(\omega_0 t + \Phi_I)$$

where,

$0 \le t \le T$
$I = 1, 2, ..., M$
$\Phi_I = (2\pi I)/M$

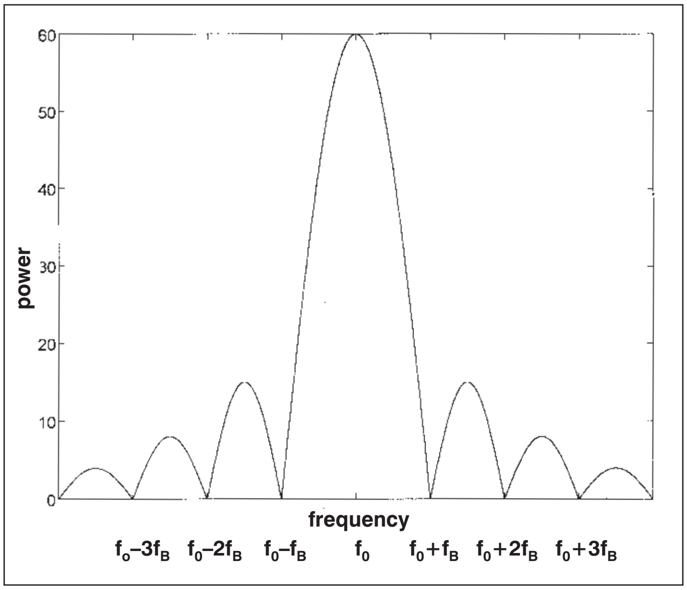
A = amplitude of carrier
ω_0 = frequency of carrier
Φ_I = phase angle of the carrier
N = number of data bits per symbol (=1 for BPSK, =2 for QPSK)
M = number of symbols = 2^N

In Binary PSK (BPSK), only one bit (binary 1 or 0) is transmitted every bit period T_B. This corresponds to a transmitted signal whose phase is either 0 or π radians respectively, with phase changes occurring at the bit rate $f_B = 1/T_B$.

In M-ary Phase Shift Keying (MPSK), N bits are combined and transmitted as a symbol. There can be a total of $M = 2^N$ different symbols, each symbol being a different combination of N bits. Each transmitted symbol is represented by a different phase of the signal. Therefore, the signal can have up to M phase changes occurring every symbol period or every T_S seconds, where $T_S = NT_b$.

For example, in Quaternary PSK (QPSK, N=2, M=4), two bits are combined to form a symbol before transmission, each symbol being represented by a different phase of the transmitted signal. Phase change occurs every symbol period. The transmitted signal can have any of four phases — $\pi/2$, π, $3\pi/4$ or 2π — depending on the symbol (00, 01, 10, 11 respectively) being transmitted.

The power spectral density or PSD (i.e. the power distribution versus frequency in the spectrum) of a PSK waveform is:



■ **Figure 2. PSD of BPSK [sin(x)/(x)]showing sidelobes.**

$$PSD_{PSK}(f) = (PT_S/2)\{\text{sinc}^2 \pi(f-f_0)T_S + \text{sinc}^2 \pi(f+f_0)T_S\}$$

where,

P = power in carrier
T_S = symbol period = bit period T_S for BPSK
f_0 = carrier frequency
$\text{sinc}(x) = \sin(x)/(x)$

In the PSD of BPSK shown in Figure 2, the spectrum is centered at the carrier frequency f_0 with zero crossings occurring at distances from f_0 of $\pm f_B$, $\pm 2\, f_B$, $\pm 3f_B$, wherever f_B is the bit rate. The main lobe of the spectrum contains 90% of the signal power with side lobes extending on to adjacent channels and falling off to zero as f moves away from f_0. The side-lobes are the primary cause of interchannel interference with adjacent channels.

A form of QPSK is used in the North American Digital Cellular standard as well as in the IS-95 Code-Division-Multiple-Access (CDMA) scheme.

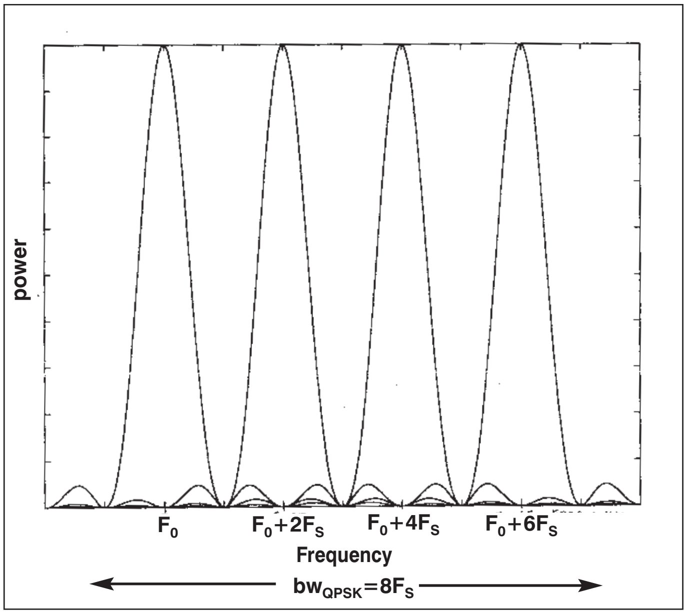
Frequency Shift Keying (FSK)

In FSK, the frequency of the carrier is changed as a function of the modulating signal (data) being transmitted. Amplitude and phase remain unchanged. FSK is represented by:

$$v_{FSK} = A\cos(\omega_I t + \Phi),$$

where,

$0 \le t \le T$ and
$I = 1, 2,...., M$

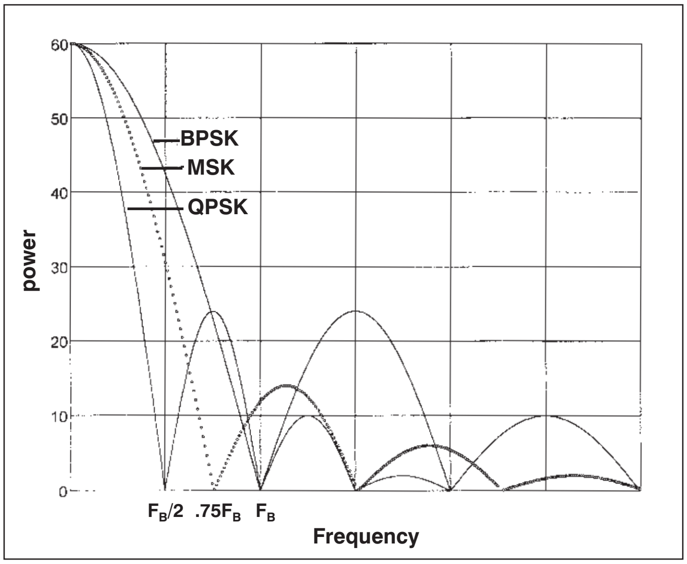


■ **Figure 3. PSD of QFSK.**

A = amplitude of carrier
ω_I= frequency of carrier
Φ = phase angle of the carrier
N = number of data bits per symbol
M = number of symbols = 2^N

Binary FSK (BFSK) is similar to BPSK, except that instead of transmitting a signal with a different phase each time a 1 or a 0 is sent, a different frequency is transmitted, ω_1 for a binary 1 and ω_2 (= $\omega_1 + \delta\omega$) for a binary 0. The power spectral density (PSD) of BFSK consists of two separate BPSK-type $\text{sinc}^2(x)$ spectra, one centered at ω_1 and the other at ($\omega_1 + \delta\omega$) respectively.

M-ary FSK is similar to M-ary PSK, except that transmitted symbols are represented by signal frequency changes instead of signal phase changes. In M-ary FSK, N bits are combined into a symbol before transmission, each symbol being represented by a separate frequency of the signal. There can be a total of M different frequencies, with each change in frequency occurring every symbol period or every T_S second, where $T_S = NT_B$.



■ **Figure 4. BPSK, QPSK, MSK Comparison.**

For example, in Quaternary FSK, the data is transmitted in symbols of two bits. Each symbol is represented by a different frequency of the transmitted signal, with each frequency change occurring every symbol period. The signal can have any of four frequencies, depending on the type of symbol (00, 01, 10 or 11) sent. The frequencies are spaced equidistant from each other and the spacing generally is even multiples of the symbol frequency, f_S.

The power spectral density of a MFSK consists of M separate $\text{sinc}^2(x)$ waveforms, the first waveform being centered at frequency ω_1, the second at ω_2, and the Mth waveform at ω_M. Figure 3 shows the PSD of QFSK.

Minimum Shift Keying (MSK)

The PSD of PSK is a $\text{sinc}^2(x)$ spectrum with the main lobe centered at the signal frequency and side-lobes of decreasing amplitudes extending on to the neighboring channels, creating interference with neighboring channels (also called spectral regrowth). This problem is so severe that the FCC requires interfering side lobes to be suppressed prior to transmission.

Spectral regrowth can be reduced by using a modulation type called Minimum Shift Keying (MSK). MSK occurs when the transmitted signal phase changes are "gradual" (as opposed to "abrupt" in QPSK, wherein phase changes occur in increments of $\pi/2$). MSK creates sidelobes that are much lower in amplitude. Smooth phase changes in MSK are accomplished by filtering the digital data before modulation. This may be accomplished by using a lowpass filter. However, the addition of a filter in the data path creates symbol distortion, and therefore, the choice of filtering must take into consideration the tradeoff between symbol distortion and sidelobe suppression. Figure 4 shows the comparison of three modulation schemes.

The Digital European Cellular Standard GSM uses a form of MSK called GMSK (Gaussian Minimum Shift Keying).

Bandwidth efficiency of PSK and FSK

Clearly, as communication systems usage increases, there is a premium placed on the available RF bandwidth over which the communications must take place. In this regard, it is important that each communication use as little bandwidth as possible, consistent with the information bandwidth (modulation rate) it must transmit. For this reason, the Federal Communications Commission often requires communication efficiency, using the term bandwidth efficiency.

Bandwidth efficiency (ρ) is defined as the transmitted bit rate (f_B) divided by the bandwidth of the signal. The bit rate is $f_B = 1/T_B$ and the bandwidth of MPSK (defined as the width of the mainlobe of the spectrum) is $bw_{PSK} = 2/T_S$, where $T_S = T_B \log_2 M$, T_B (= $1/f_B$) is the bit period of BPSK and M is the number of symbols. As M increases, the bandwidth of the signal decreases by a factor of $\log_2 M$. For example, QPSK occupies half the bandwidth of BPSK.

The bandwidth efficiency of MPSK is:

$$\rho = f_B/bw_{MPSK} = (\log_2 M)/2$$

This efficiency, ρ, increases with increasing M. Table 1 shows ρ as a function of M for MPSK:

M	2	4	8	16
ρ (bps/ Hz)	.5	1	1.5	2

■ **Table 1. Bandwidth efficiency, ρ, versus M for M-ary PSK.**

On the other hand, since the PSD of MFSK contains up to M separate $\text{sinc}^2(x)$ spectra, the bandwidth required to recover a MFSK signal is the sum total of the bandwidth of each $\text{sinc}^2(x)$ spectrum.

By selecting the channel frequencies as successive even multiples of the symbol frequency f_S (e.g. for QFSK, the four frequencies are kf_S, $(k+2)f_S$, $(k+4)f_S$ and $(k+6)f_S$ as shown in Figure 3), the bandwidth required to recover FSK is:

$$bw_{MFSK} = M(2)/T_S$$

where,

$$T_S = T_B \log_2 M$$

Therefore, the bandwidth efficiency of MFSK is:

$$\rho = r_B/bw_{MFSK} = (\log_2 M)/(2M)$$

For BFSK (M = 2) and QFSK (M =4), the bandwidth efficiency of MFSK remains the same. Thereafter, ρ decreases as M increases.

M	2	4	8	16
ρ (bps/ Hz)	.25	.25	.1875	.125

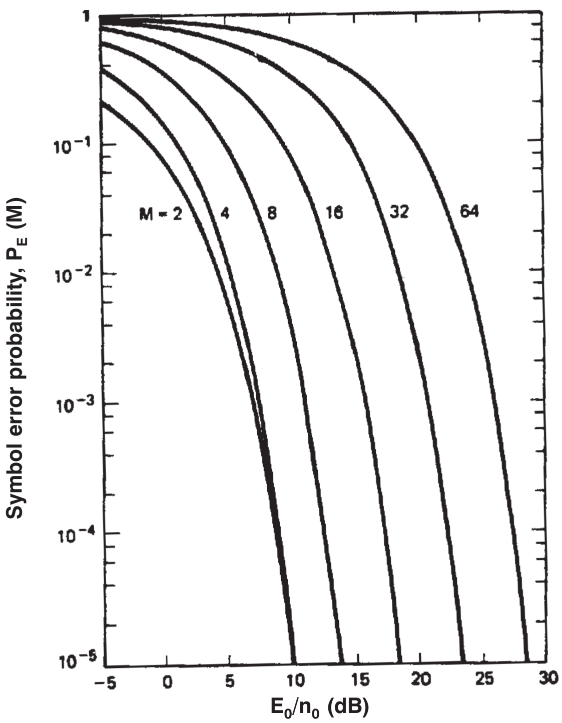
■ **Table 2. Bandwidth efficiency, ρ, versus M in MFSK.**

MFSK occupies considerably more bandwidth than MPSK and is therefore not used where spectral resources are scarce.

Probability of Bit Error (P_E)

A measure of performance in digital communication systems is the Probability of Bit Error (P_E) at the receiver or the probability that any given bit being received will be in error.

Naturally, the lower the P_E, the better the performance of the receiver all other things being equal. P_E is mainly a function of signal-to-noise ratio (or E_B/N) at the receiver and is given as:



■ **Figure 5. Symbol error probability for coherently detected multiple phase signalling.**

$$P_E(E_B/N) \propto Q(SQRT(E_B/N))$$

Q(x) is called a complementary error function of x. It decreases exponentially as x increases; that is, P_E decreases with increasing signal power.

P_E also can be changed by varying M. For MPSK, increasing M causes P_E to degrade, whereas P_E improves with M for MFSK.

Figures 5 and 6 show P_E as a function of the signal to noise ratio (SNR) for PSK and FSK respectively, for different values of M.

The PSD of a digital modulated waveform is a $\text{sinc}^2(x)$ spectrum centered at the channel frequency with sidelobes extending on to adjacent channels. The width of the main lobe decreases as M increases. Therefore, for MPSK, the bandwidth decreases with M. However, for MFSK, the bandwidth increases with M; because, although the width of the main lobe decreases, the number of $\text{sinc}^2(x)$ spectra increase as the number of bits combined and transmitted as symbols (N) increase. MFSK occupies more bandwidth than MPSK primarily because the PSD of MFSK consists of multiple MPSK spectra.

The quality of reception is judged by the Probability of Symbol Error (P_E) at the receiver, which decreases as the signal-to-noise ratio at the receiver increases. PE is also a function of M.

Signal power, bandwidth and circuit complexity are key considerations when selecting a modulation technique in digital communication systems. MPSK is used wherever the available bandwidth is scarce, and the bandwidth can be further reduced by combining more bits into a symbol before transmission. However, increasing N makes the circuit more complex and in the case of MPSK, also increases the power requirements of the channel. ■

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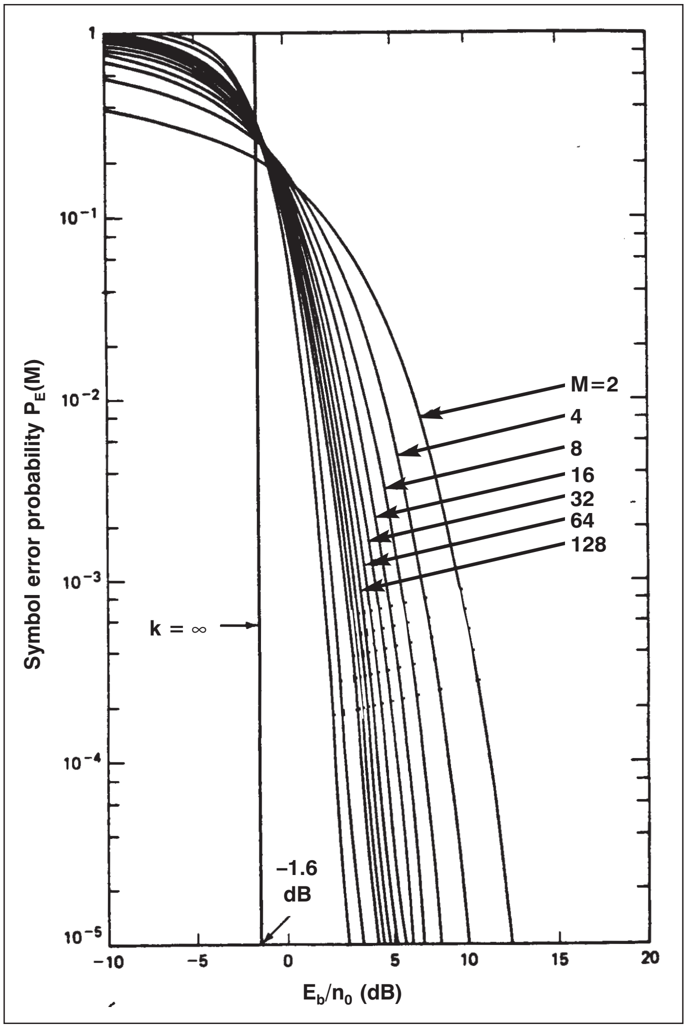

■ **Figure 6. Symbol error probability for coherently detected MFSK.**