

# Multipath Fading Measurement in Real Time

*Measuring multipath signals on the fly between a moving mobile transmitter and a fixed base station is too much for conventional spectrum analyzers. The authors describe a bank-of-filters system for multipath evaluation in real time.*

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A common problem with mobile communications systems in urban environments is the signal fading caused by reflections of the transmitter signal from nearby structures. In a system such as a digital cellular telephone having a stationary repeater and mobile subscribers, two or more multipath signals can arrive at the receiver out of phase, nearly cancelling the net signal received and interrupting service.

Naturally, it is desirable for the service provider to know these locations in order that corrective actions can be taken. Furthermore, since the environment often changes through new construction and other factors, the transmission area should be checked periodically to verify the locations and severity of known "blind spots" and to ascertain the presence of any new ones.

This article describes a technique for monitoring multipath fading "on the fly." The method makes measurements of multipath fading in an active system without the need for special test signals. To do so it takes advantage of the wide band nature of phase shift keyed (PSK) signals and the real time spectral analysis possible with current instrumentation based on digital signal processing. The technique gives a readout indicat-

ing the approximate location of the reflecting structure, as well as the relative amplitude of the reflections.

### Background

To examine the nature of multipath fading, first consider the case of a single, unmodulated carrier. This case can be extended to the general case on the assumption that the transmission medium is linear, usually a reasonable approximation. The sum of a carrier and its delayed and attenuated reflection can be written as shown in Equation 1.

$$(1) \quad x(t) = \cos(\omega t) + \alpha \cos(\omega(t - \tau))$$

The symbol  $\omega$  is the radian frequency of the carrier and  $\alpha$  and  $\tau$  represent the attenuation and delay of the reflected signal, respectively. The power spectrum of this signal will consist of a line at  $f = \omega/2\pi$  with a factor including the attenuation due to the reflected signal.

$$(2) \quad X(f) = \delta(2\pi f - \omega) + \alpha e^{-j2\pi f \tau} \delta(2\pi f - \omega)$$

$$(3) \quad = \delta(2\pi f - \omega) (1 + \alpha e^{-j2\pi f \tau})$$

The Dirac delta function  $\delta(\phi)$  is used to represent the discontinuity consisting of a finite amplitude spectral line at a zero bandwidth carrier frequency. The second term in Equation 3 results from summing the direct and reflected signals and has the general shape of a cosine function. Also note that this term is independent of the carrier frequency  $\omega$  so that, in the general case, the form of this term will not change.

Since standard instruments measure the power spectrum, it is necessary to multiply Equation 3 by its complex conjugate to obtain a mathematical expression for what will be seen on a spectrum analyzer. This is shown, after some manipulation, in Equation 4.

$$(4) \quad P_x(f) = X(f)X(f)^*$$

$$= (1 + \alpha^2 + 2\alpha \cos(2\pi f \tau)) \delta(2\pi f - \omega)$$

Equation 4 shows the cosine relationship explicitly, as well as the weighing caused by the power of the reflection ( $\alpha$ ). The peak-to-peak amplitude of the cosine wave term in Equation 4 is 4. If the carrier in our example were modulated, the Dirac delta term in Equation 4 would be replaced by a sum of frequency components representing the Fourier transform of the RF spectrum of the transmitted signal. In this case, the spectrum is represented as shown in Equation 5 with  $S(f)$  indicating the power spectrum of the transmitted signal.

$$(5) \quad P_x(f) = (1 + \alpha^2 + 2\alpha \cos(2\pi f \tau)) S(f)$$

Generally, there are more reflections present than one and, therefore, several additional terms of the form shown in Equation 5. There also will be cross terms but these are assumed to be negligible, being limited in amplitude to the amplitude of the smallest reflection. The time delay and attenuation of multipath signals can be found simply by finding the spectrum of the power spectrum of the signal containing multipath. This yields the frequency and power of the sine wave components which are directly related to the multipath parameters.

### Real-time Spectral Analysis

Obtaining the power spectrum of a noise-like modulation such as a PSK signal in a dynamic environment is not a trivial task. A typical spectrum analyzer uses a swept local oscillator and a fixed resolution filter to estimate the spectrum of an RF signal. The resulting spectrum is a function of time as well as frequency, consequently video filtering or display averaging must be employed. But this filtering extends measurement time considerably, limiting the technique's effectiveness to those applications wherein both the transmitter and receiver are stationary. On the other hand, measurements in a moving vehicle require a wholly different type of instrumentation.

Our approach uses a bank of 800 band pass filters to divide the measurement spectra into intervals of up to 10 MHz. In the bank-of-filters method (Figure 1) the input signal is simultaneously and continuously applied to each filter. (In digital signal processing literature, the bank of filters is often referred to as "the [bank of] polyphase filters.") Each filter has a narrow resolution bandwidth. This is similar to the RBW filter of a spectrum analyzer. The difference in our approach is that there are multiple filters in parallel, and each filter's bandwidth is contiguous with those adjacent to it. As a result, the ensemble of output signal levels of the filters itself represents a spectral display of the input signal.

Implementing such a filter bank for practical application requires creation of enough filters to provide adequate resolution over the frequency span of interest. Each filter must have a flat passband for accurate spectral power measurements. Each filter also must have sharp skirts (roll off) both for high resolution as well as to provide high out-of-band rejection for good dynamic range. Additionally, the filters' transmission characteristic must be replicated precisely for equal and adjacent placement

across the frequency span to be analyzed.

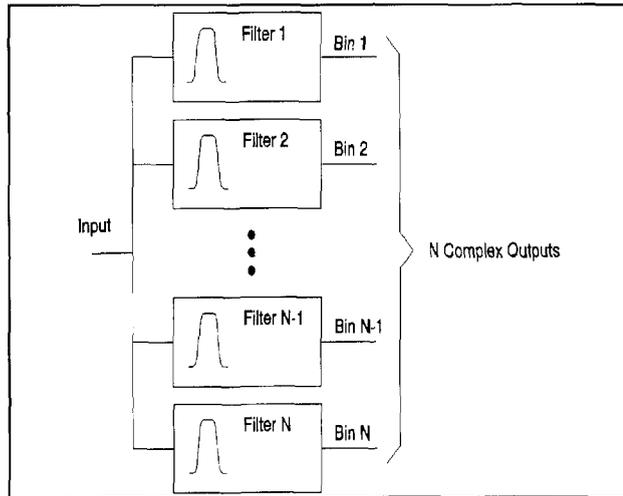


Figure 1. A bank of parallel passband filters provides simultaneous and continuous monitoring of a span of frequencies.

To meet these stringent filterbank requirements, a special digital filter was designed. This filter is a finite impulse response (FIR) filter with the shape shown in Figure 2.

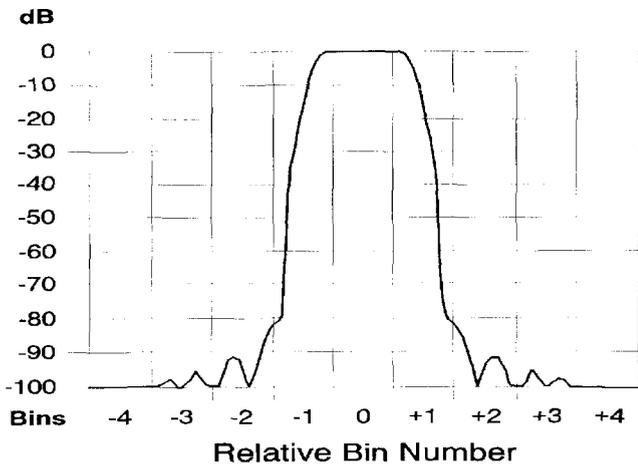


Figure 2. The bank-of-filters technique was implemented using a special filter shape that is extremely flat over the bin width and with very steep skirts outside of the bin.

Notice in Figure 2 that the filter response is very flat over what is referred to as its frequency bin width (Relative Bin Number 0 in Figure 2). A high degree of flatness is necessary to represent accurately the power of any spectral element occurring within the filter's bin width. For our filters the bin flatness is 0.05 dB. From this flat pass band, the skirts drop sharply from 0 dB to -80 dB in an interval of less than a bin width.

This filter shape, when replicated into a filter bank by frequency shifting the digital filter coefficients, provides the following measurement performance characteristics

listed in Table 1.

<b>Amplitude accuracy:</b>	<b>-/+ .15 dB</b>
<b>Sensitivity</b>	<b>-150 dBm/Hz</b>
<b>Display dynamic range</b>	<b>84 dB</b>
<b>Residual response</b>	<b>&lt;-70 dB below maximum input</b>

Table 1. Real time multipath measurement specifications obtainable using a bank-of-filters system having filters with 0.05 dB flatness and 80 dB rejection within a bin width.

### Setting Span, CF, and Resolution

The span and center frequency (CF) selection is effected using a five-stage quadrature digital down converter (DDC) that precedes the filter bank. The locations of the DDC and filter bank relative to each other and the overall system architecture are shown in Figure 3.

The input signal is, first, digitized then fed into the DDC for analysis. The DDC's five stages consist of quadrature local oscillators and mixers followed by lowpass (anti-aliasing) filters. Each of the low pass filters is a pass-through filter ( $\div$  by 1), or a decimator ( $\div$  by 2, 5, or 10) according to the span selected. This allows span selection from 100 Hz to 10 MHz in a 1, 2, 5 sequence. Additionally, for spans narrower than 10 MHz, the span's center frequency can be tuned in increments finer than a bin width.

The essence of the DDC process is that the input signal is frequency down converted so that the selected center frequency corresponds to the center of the filter bank and the span corresponds to the central displayable bins. This allows a fixed filter bank to be used for all spans and center-frequency selections that can enter the receiver.

Changing spans effects a zoom function into the selected center frequency area. With a fixed transform length, reducing the span increases the displayed frequency resolution. For example, using a transform length of 1024 bins and a 1-MHz span provides a bin resolution of 1.25 kHz (1-MHz span covered by 800 display bins). With the same transform length, spanning down to 100 Hz results in a resolution of 0.125 Hz.

Frequency resolution can also be varied by changing transform length. With a 1-MHz span for example, changing the transform length from 1024 to 256 changes bin resolution from 1.25 kHz to 5 kHz.

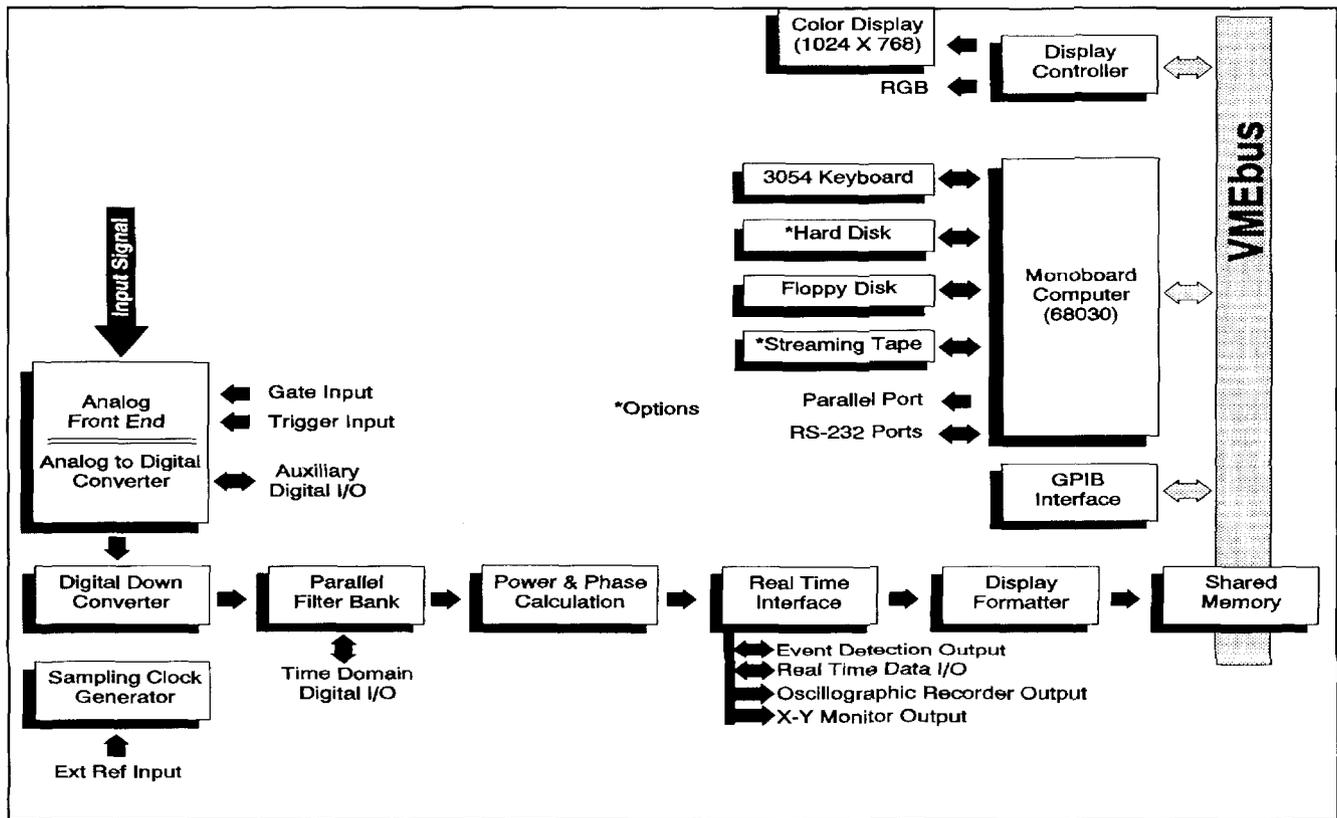


Figure 3. Block diagram of the bank-of-filters measurement system.

The major advantage of reducing transform length is that the number of bins to be processed is reduced. This, in turn, allows a correspondingly faster frame update rate, providing increased time resolution.

### Update Rate and Time Resolution

Frame update rate and time resolution are key issues in spectrogram displays. A real time spectrogram derived from the bank-of-filters measurement described herein is sometimes called a waterfall display, and consists of a time series display of spectral frames. We chose to display frequency on the horizontal axis and time on the vertical axis. This presents the spectrogram display as a series of horizontal lines with each line representing a spectral frame in time. The spectral power in each bin is represented on the display by color coding with 10 stratified bands (see photo of the display on the front cover of this issue).

The time between each frame is the time resolution of the display and depends upon the frame update rate (how rapidly new spectral frames can be captured). Figure 4 illustrates this concept.

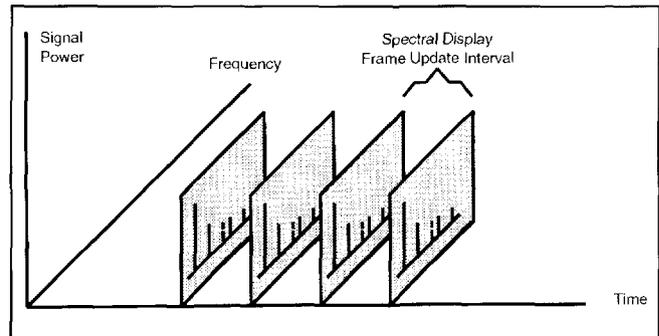


Figure 4. The frame update rate is the time interval between spectral frame captures, in this system example, frame update rates recur in as little as 12.5  $\mu$ s.

### Measurement Technique

This measurement technique is ideal for capturing wideband signals found in PSK. The measurement system consists of a wide frequency range spectrum analyzer tuned to the digital cellular telephone frequency and with RF down conversion to the input frequency of the bank-of-filters digital spectrum analyzer. With this approach, a "snapshot" of a power spectrum can be obtained in only 200 microseconds. An average of 50 spectra can be computed continuously every 10 milliseconds. In the cover photo a split display from the system is shown.

The upper display is a spectrogram, revealing the power

spectrum of a signal as a function of time. The lower display shows a single spectrum of a signal with multipath in white and the delay spread in red.

The delay spread is computed by first obtaining a sample of the power spectrum without multipath. This spectrum is stored and used as a reference. The spectrum of PSK signals is generally time invariant so that it can be sampled with sufficient smoothing and then compared to a spectrum measured at a later time.

### *Test Conditions*

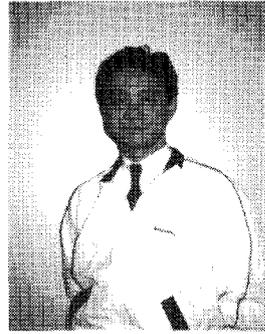
A vehicle containing the measurement system is driven through the area under test and data are collected using the bank-of-filters system. Our system can capture up to 4000 spectra in real time, corresponding to 40 seconds of real time measurements with a 10 millisecond remeasurement interval. The captured data then can be processed to determine the delay spread.

The spectrum containing multipath are weighted by the cosine functions discussed earlier. A Fast Fourier Transform (FFT) of this spectrum is taken in order to obtain the delay spread. The resolution of the FFT is reduced by the inherent "windowing" caused by the roll off in the spectrum of the original signal.

To improve frequency resolution, the signal spectrum is "whitened" by subtracting the reference spectrum from the test spectrum. The result is the sum of the cosine functions from the mutipath. The FFT of this spectrum gives the delay spread. This technique was used to generate the red trace in the cover photo.

Further improvement can be made to the time resolution of the delay spread by taking advantage of the sinusoidal nature of the multipath spectrum. Parametric spectrum estimation techniques such as Maximum Entropy can be employed to give resolution improvements on the order of 10 to 1.

Measuring multipath fading in mobile systems is extremely difficult, if not altogether impractical, without the bank-of-filters measurement system, wherein, for PSK signals, wide bandwidth DSP technology can be used to give real-time measurements of multipath spectra. Post processing techniques such as whitening of the signal spectrum greatly improve the accuracy and sensitivity of this technique.



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### *Editor's Note*

*The authors' firm markets a system, designated the Model 3054 DSP, which embodies the measurement techniques described in this article.*