Simulating Rayleigh Fading with MATLAB

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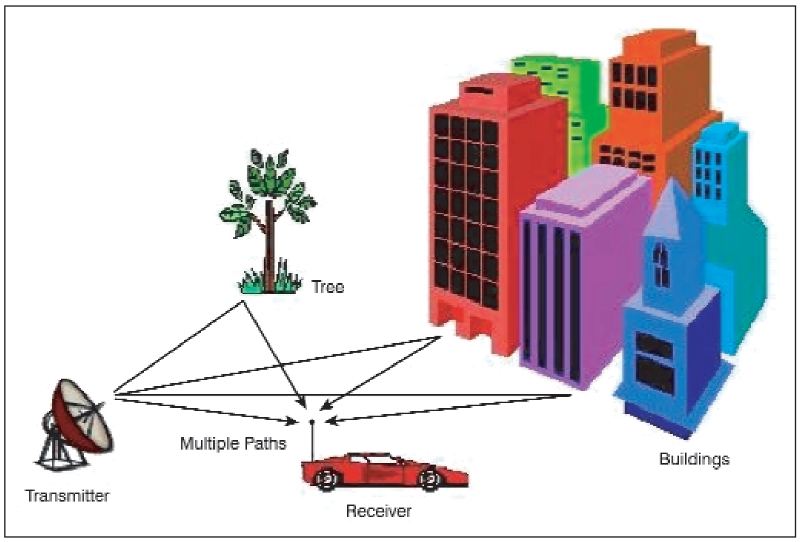
In mobile communications the continuous change of transmitter and receiver positions produces a variation in signal paths.

The fluctuations produced by diffraction, reflection and Doppler effect cause fast amplitude fading and signal delays. Signals arrive at the receptor from multiple paths, such as signal reflections and refractions on the environment (buildings, trees, vehicles). Statistically, the signal fading is described by Rayleigh's distribution. This article shows how to simulate signal fading using MATLAB and how frequency and mobile speed modify the attenuation pattern [1].



▲ **Figure 1. Rayleigh's multiple paths.**

Theory

Jakes's model [2] can be used to describe the signals' behavior.

- A finite number of field components, called N, arrive at the receiver with the amplitude:

  $$\frac{E_0}{\sqrt{N}}$$

  The azimuthal angles are uniformly distributed as:

  $$\alpha_n = \frac{2 \cdot n}{N} \cdot n \qquad n = 1,2,3,\ldots\ldots,N$$

- The receiver moves with speed v; each field component arrives at the receiver with a frequency shift produced by the Doppler effect:

  $$W_n = W_m \cdot \cos(\alpha_n)$$

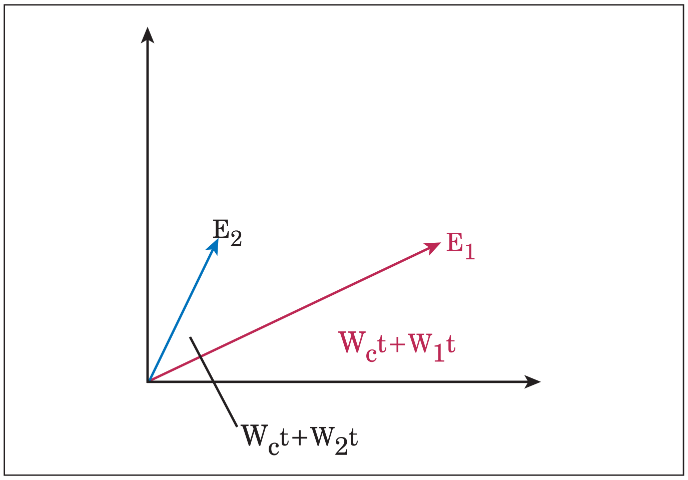
  $$W_m = W_c \cdot \frac{v}{c} \quad \text{maximum frequency shift}$$

  $$\mathrm{W_c} = 2 \cdot \pi \cdot f_c \quad \text{carrier frequency}$$

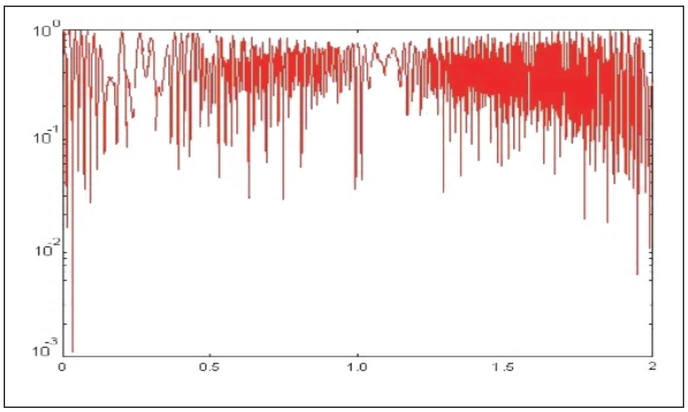
- The received signal can now be written as follows:

  $$E(t) = \frac{E_0}{\sqrt{N}} \cdot \sum_{n-1}^{N} \cos\left[\left(W_c + W_n\right) \cdot t + \phi_n\right]$$

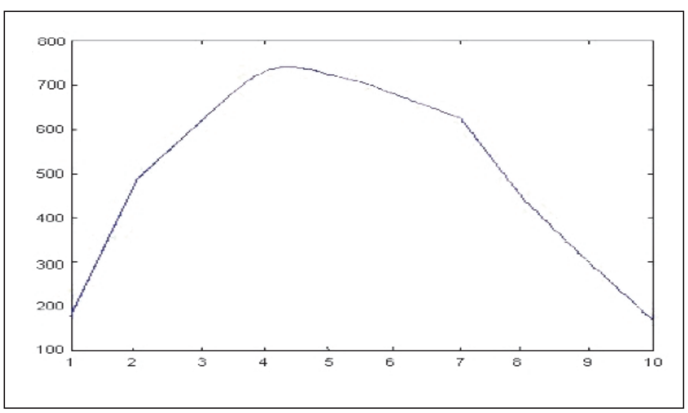
  ϕ_n : n - path random phase



▲ **Figure 2. Vectors graph showing two of the N components.**

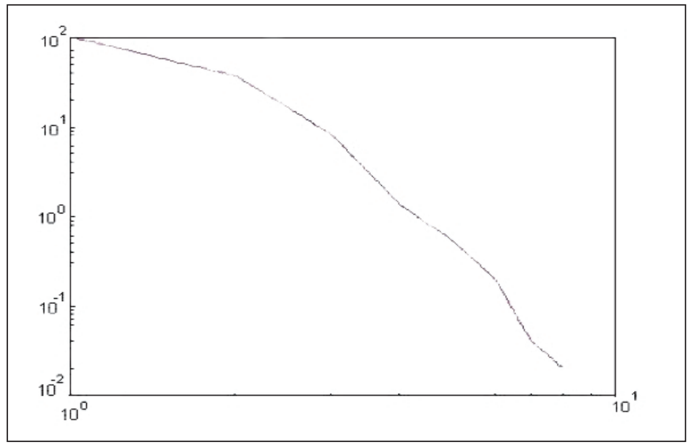


▲ **Figure 3. Rayleigh's fading (*f* = 900 MHz – *v* = 30 km per hertz). The x-axis shows time in seconds. The y-axis shows the signal level (standardized)**

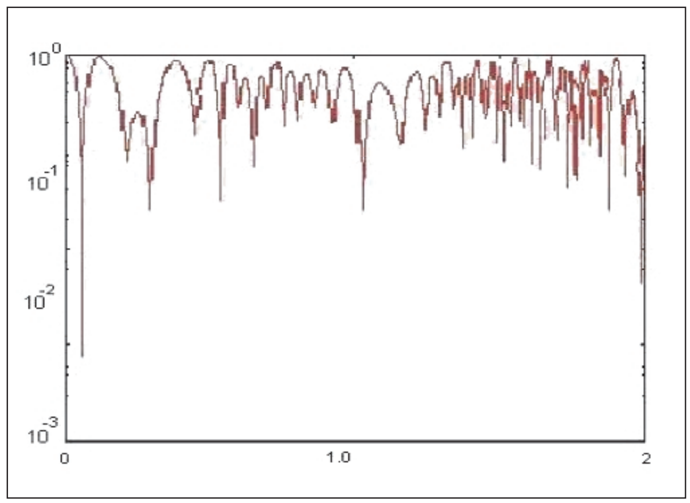


▲ **Figure 4. A typical Rayleigh's distribution (approximate).**

$E(t)$ is the sum of the random signals and its components are statistically independent. We can also use a vector graphic to represent the $E(t)$ equation. Two of the N components are shown in Figure 2.



▲ **Figure 5. Probability of exceed of a given attenuation. This indicates the probability (in percentage) of exceed of a given attenuation (in dB).**



▲ **Figure 6. The number of fadings decreases as the receiver speed tends to 0 km per hertz. The same effect occurs for smaller values of frequency. This graph shows Rayleigh's fading for *v* = 5 km per hertz, as well as shows how the receiver speed modifies the fading pattern.**

- The received field can be written as:

$$E(t) = \mathrm{Re}\left\{T(t) \cdot e^{jWct}\right\}$$

$$T(t) = \frac{E_0}{\sqrt{2 \cdot N + 1}} \cdot \left[xc(t) + j \cdot xs(t)\right]$$

$$xc(t) = 2 \cdot \sum_{n-1}^{N} \cos(W_n \cdot t) \cdot \cos(\phi_n) + \cos(W_m \cdot t)$$

$$xs(t) = 2 \cdot \sum_{n-1}^{N} \cos(W_n \cdot t) \cdot \sin(\phi_n)$$

$$\phi_n = \frac{\pi \cdot n}{N + 1}$$

where $T(t)$ is the complex envelope; *xs* and *xc* are its projection on the real and imaginary axis respectively.

- The envelope $T(t)$ has a Gaussian distribution for large values of N with phase uniformly distributed. Finally, we can verify that the module of $T(t)$ is distributed according to Ray-leigh's law.

Conclusion

This article has shown how to describe the signal behavior for mobile communications, and how signal behavior can be simulated using MATLAB.

The program can be written using MATLAB editor and either receiver speed or frequency can be changed and the different fading patterns can be observed. ■

References

1. B. Keiser and E. Strange, *Digital Telephony and Network Integration*, New York: John Wiley & Sons, Inc., 1995.

2. W. M. Jakes, *Microwave Mobile Communications*, New York: John Wiley & Sons, Inc., 1974.

3. D. Etter, *Solucion de problemas de Ingenieria con Matlab*, Engle-wood Cliffs, NJ: Prentice Hall, 1997.

Author information

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