

# Lumped Element Quadrature Hybrid Provides Supplementary Low Pass Filtering

By Richard M. Kurzrok, PE  
RMK Consultants

A quadrature hybrid, also called a branch line directional coupler, uses quarter-wave transmission lines for both the main lines and branch lines, as shown in Figure 1. Upon replacing each quarter-wave transmission line with its  $\pi$  equivalent, the hybrid is implemented using artificial lines, as shown in Figure 2. This technique has also been used for the one and a half wavelength hybrid ring [1]. The replacement of transmission lines by lumped circuit elements results in substantial size reduction. This enhances inclusion of the hybrid in integrated circuits, which could include active devices.

## Design equations and circuit elements

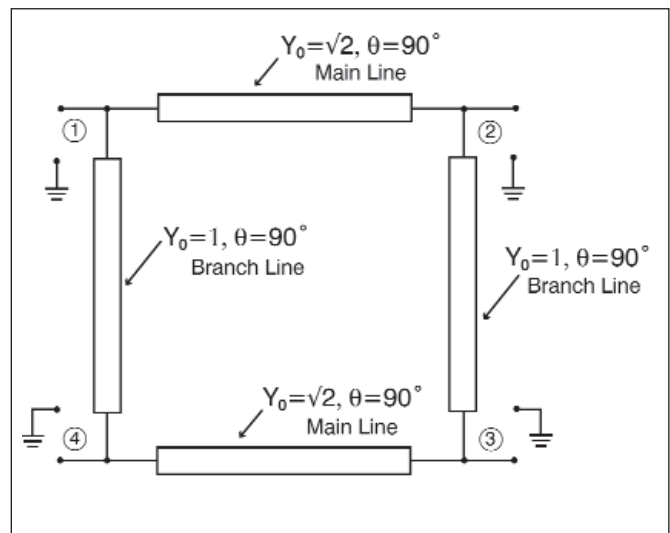
Design equations for the hybrid are shown in Equation (1). As an illustrative example, a hybrid has been designed for a center frequency of 60 MHz and a characteristic impedance of 50 ohms. Circuit element values are summarized in Table 1.

$$L = \frac{Z_0 \times 10^3}{2\pi f} \text{ nH} \tag{1}$$

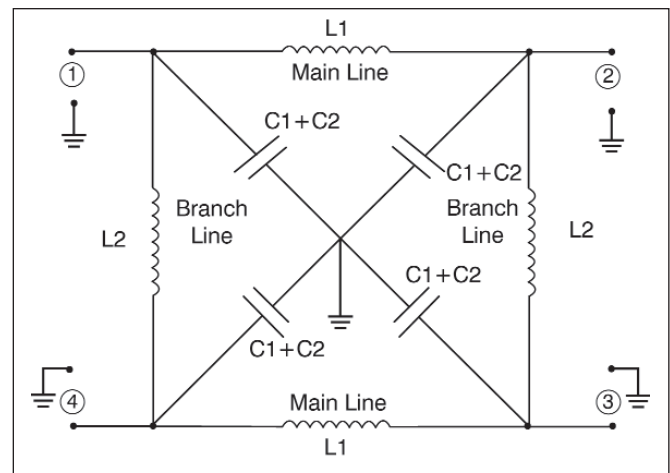
$$C = \frac{10^6}{2\pi f Z_0} \text{ pF}$$

where

- $Z_0$  = source and load impedances in ohms;
- $f$  = hybrid center frequency in MHz;
- $L$  = series inductance; and
- $C$  = shunt capacitance.



▲ Figure 1. Diagram of a quadrature hybrid with transmission lines.



▲ Figure 2. Diagram of a quadrature hybrid with artificial lines.

Parameter	Main Lines	Branch Lines
$Y_0$ – siemens	0.02828	0.02
$Z_0$ – ohms	35.36	50
L – nH	L1 = 93.8	L2 = 132.63
C – pF	C1 = 75.02	C2 = 53.05
Total Shunt Capacitance = 75.02 + 53.5 = 128.07 pF		

▲ **Table 1. Circuit element values for quadrature hybrid ( $Z_0 = 50$  ohms and  $F = 60$  MHz).**

x	f	P2 (dB)	P3 (dB)	P4 (dB)
0.1	6	6.2	6.1	6.2
0.3	18	7.7	6.7	7.2
0.5	30	9.3	6.5	8.2
0.7	42	9.2	4.9	8.5
0.90	54	4.1	3.1	13.0
0.92	55.2	3.7	3.1	14.7
0.94	56.4	3.4	3.1	17.0
0.96	57.6	3.2	3.0	20.5
0.98	58.8	3.1	3.0	26.6
1.0	60	3.0	3.0	52.2
1.02	61.2	3.1	3.0	25.9
1.04	62.4	3.2	3.1	20.1
1.06	63.6	3.4	3.1	16.8
1.08	64.8	3.6	3.1	14.6
1.10	66	3.9	3.2	12.9
1.3	78	5.1	5.3	7.2
1.5	90	7.1	11.1	9.1
1.7	102	12.5	19.9	14.9
1.9	114	17.3	27.5	19.9
2.1	126	21.2	33.6	24.0
2.3	138	24.4	38.8	27.3
2.5	150	27.2	43.4	30.1
2.7	162	29.7	47.4	32.6
2.9	174	31.9	51.1	34.9
3.1	186	34.0	54.4	36.9
x = normalized frequency variable, f = frequency in MHz				

▲ **Table 2. Transmission responses from Port 1 to Ports 2, 3 and 4 for the quadrature hybrid where  $Z_0 = 50$  ohms,  $f = 60$  MHz and unloaded  $Q = 150$ .**

## Transmission responses

Predicted transmission responses between a port 1 input and ports 2, 3 and 4 outputs are shown in Table 2. These computations utilize the classic method of analysis for symmetrical four port networks (2). The hybrid is bisected and reduced to a two port network, which is analyzed via even and odd mode excitations. Unloaded  $Q$ s of 150 have been assumed for all inductors.

## Conclusion

The quadrature hybrid, using only two branch lines, is limited in useful bandwidth to about five percent. Wider bandwidths can be obtained using more than two branch lines [2, 3]. At frequencies above 150 MHz, appreciable supplementary low pass filtering is achieved for the nominal 60 MHz hybrid. Lumped element hybrids [4] have become useful in monolithic microwave integrated circuits (MMICS). ■

## References

1. R.M. Kurzrok, "Design Technique for Lumped-Circuit Hybrid Rings," *Electronics*, May 18, 1962.
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3. G. Matthaei, L. Young and E.M.T. Jones, *Microwave Filters, Impedance Matching Networks, and Coupling Structures*, New York: McGraw-Hill, 1962: 809-835.
4. S. J. Parisi, "A Lumped Element Rat Race Coupler," *Applied Microwave*, August/September 1989: 130-135.

## Author information

Richard M. Kurzrok, PE, is an independent consultant specializing in filters and equalizers from baseband through microwave frequencies. He may be reached at RMK Consultants, 82-34 210th Street, Queens Village, NY, 11427-1310; by telephone at 718-776-6343; by fax at 718-776-6087; or by e-mail at rmkconsulting@aol.com.