Polynomial Model of Blocker Effects on LNA/Mixer Devices

An analysis of interference effects on small-signal gain and noise figure

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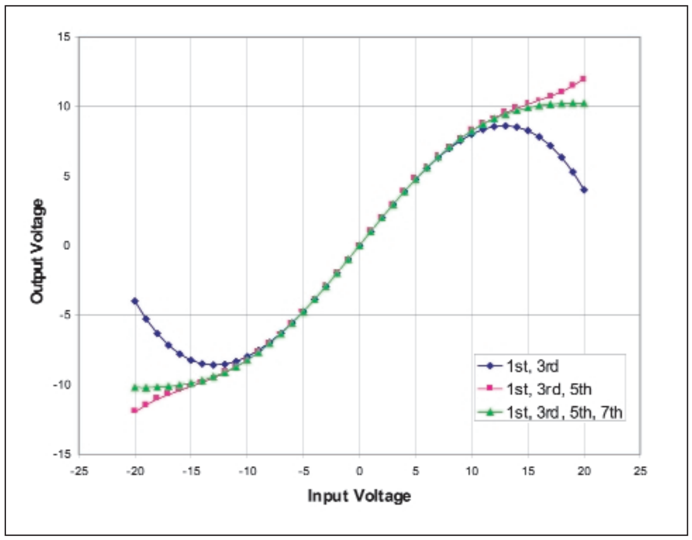
In designing today's wireless handset receivers, it is important to maximize both receive sensitivity and resistance to undesired signals, also called "interferers" or "blockers." Receiver design begins with the calculation of budgets for noise figure and linearity, usually facilitated by a spreadsheet. Although it is relatively easy to find the cascaded noise figure (NF) and 1 dB compression point (P_{1dB}) using a spreadsheet calculation, it is often not clear how to use these to predict the actual performance of the receiver in the presence of a large blocker.

A reasonably accurate prediction may instead require an inconvenient co-simulation of the system with circuit models embedded. At the level of cascade calculations rather than simulation, a simpler approach can be used.

For a digital cellular system such as Global System for Mobile Communications (GSM), well-specified blocking tests include both inband and out-of-band interferers. In these tests, the receiver front end must be able to reject the blocker while amplifying the desired signal, without exceeding the maximum allowed bit error rate. For out-of-band blockers, much of the rejection comes from a receive-band filter placed in front of the low-noise amplifier (LNA). Some designs also place a similar filter after the LNA and preceding the mixer, while others use an image-reject mixer. In the latter case, the LNA/mixer combination is often implemented as a single IC and must exhibit particularly good blocker resistance, since there is only one receive-band filter placed ahead of it.



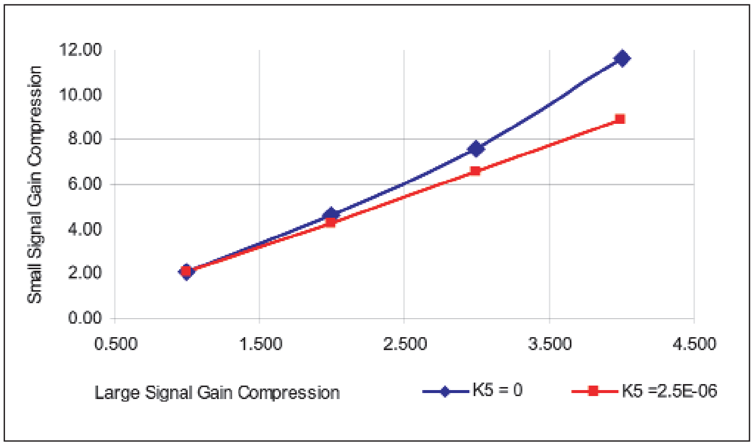
▲ **Figure 1. Polynomial compression models.**

This article discusses the effect of blockers on LNA/mixer performance. The objective is to produce a model based on empirical measurements of how an LNA/mixer's small-signal gain and noise figure degrade in the presence of large blocking signals. Once such a model is established, it can be used to accurately predict the amount of receiver desensitization resulting from varying blocker levels. The LNA/Mixer device used as an example is the Conexant RF212 dual-band image-reject downconverter for GSM.

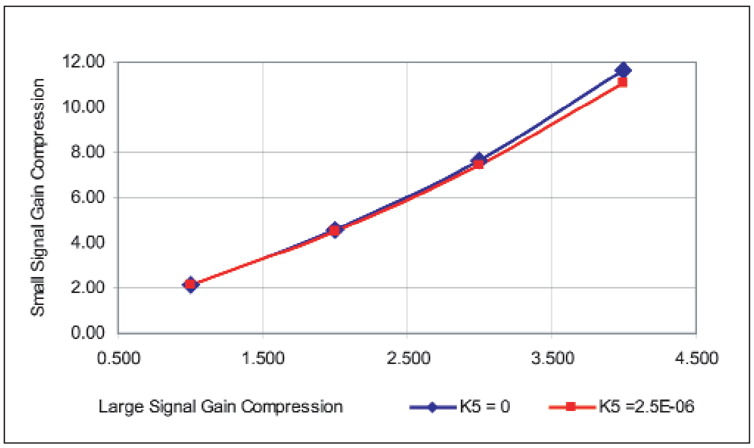
Gain compression

Typically, gain compression is only modeled as a third-order behavior, that is,

$$V_0 = k_1 V_{in} + k_3 V_{in}^3 \quad (1)$$



▲ **Figure 2. Small signal gain compression, with k_3 = .0025.**



▲ **Figure 3. Small signal gain compression, with k_3 = .0050.**

Gain compression, however, is generally a saturation-type function that cannot be completely described by the third-order component alone, which causes a downward turn of the gain curve. Figure 1 illustrates how the addition of higher order terms can produce a curve that saturates. To truly model a saturation condition would require an infinite number of terms, but a few terms, as shown in Figure 1, are sufficient to produce an adequate model. In all cases, the model cannot be used above the point where it strays from true saturation.

With first, third, fifth and seventh terms:

$$V_0 = k_1 V_{in} + k_3 V_{in}^3 + k_5 V_{in}^5 + k_7 V_{in}^7 \qquad (2)$$

To obtain the nonlinear gain, Equation (2) is divided by the input voltage.

$$Gain = k_1 + k_3 V_{in}^2 + k_5 V_{in}^4 + k_7 V_{in}^6 \qquad (3)$$

Note that the gain equation has only even-order terms. An equation of this type is particularly useful when V_{in} is a large blocker and the gain is the small-signal voltage gain in the presence of this blocker. With V_{in} always representing the blocker, the coefficients will be different for the blocker gain and the small-signal gain, since the small signal always is compressed faster than the large signal (see Appendix A).

If the transfer function model is limited to third-order and fifth-order non-linearity and the coefficients are defined as positive quantities, we can write the following equation for the large signal gain (LSG):

$$LSG = k_1 - \frac{3}{4} k_3 \left(V_{\text{Large}}\right)^2 + \frac{5}{8} k_5 \left(V_{\text{Large}}\right)^4 \qquad (4)$$

where V_{Large} is the peak amplitude of the large signal blocker. Similarly, the small-signal gain (SSG) is

$$SSG = k_1 - \frac{3}{2} k_3 \left(V_{\text{Large}}\right)^2 + \frac{15}{8} k_5 \left(V_{\text{Large}}\right)^4 \qquad (5)$$

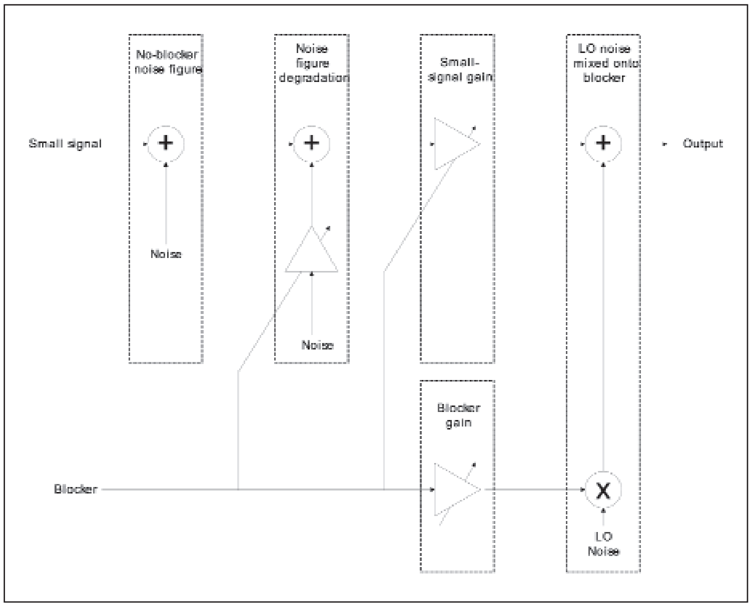
Note that the second-order term is doubled for the small signal gain, while the fourth-order term is tripled. The k_3 term causes the small signal to be compressed faster than the large signal; the k_5 term pulls the gain curve back up and keeps it from turning downward. Even though this term is tripled for the small signal gain, it is generally not enough to keep the k_3 term from causing the faster compression.

Figures 2 and 3 give some example results of compressions that occur according to Equations (2) through (5), where k_1 is unity. In these examples, two things become noticeable. First, when there is only a third-order term, the relation between SSG and LSG compression is independent of the third-order term. For instance, a compression of 1.0 dB in LSG causes a compression of about 2.1 dB in SSG, for either case of k_3, as long as $k_5 = 0$. Second, the fifth-order term has only a small influence on the relation between SSG and LSG compression until the amount of compression becomes large. This is why compression is often modeled using third-order non-linearity only.

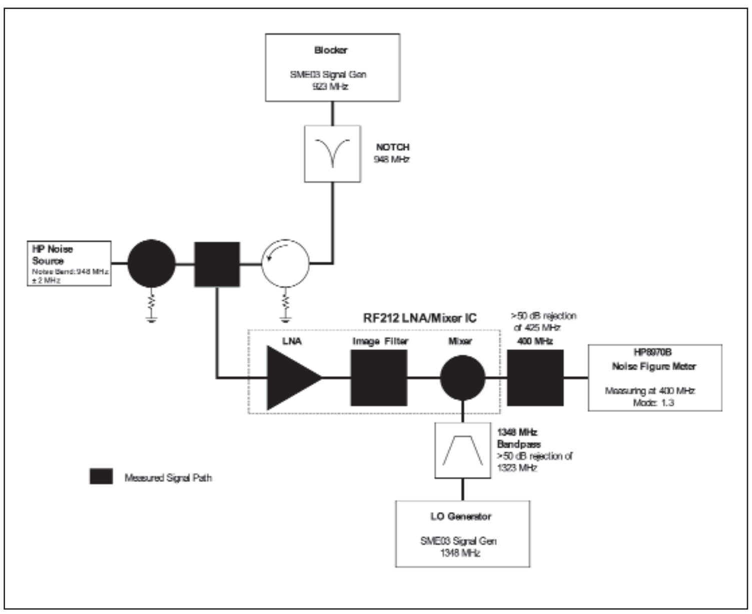
Our approach determines the coefficients of the SSG compression by measurement and curve fit, rather than assuming they keep the precise relationship with the LSG coefficients shown in Equations (4) and (5). The noise figure increase that occurs is then measured and related to the small signal gain.

System gain and noise model

Figure 4 shows how the blocker affects the gain and noise of the system. Based on Equation (5), the blocker amplitude affects the small-signal voltage gain, as



▲ **Figure 4. Model for blocker effects.**



▲ **Figure 5. Test setup to measure blocker effects.**

shown by

$$SSG = a_0 - a_2(\text{Blocker})^2 + a_4(\text{Blocker})^4 \qquad (6)$$

where
a_0 takes the place of k_1,
a_2 takes the place of 3/2 (k_3) and
a_4 take the place of 15/8(k_5).

The three noise contributors are the basic noise figure of the system, the noise figure degradation due to the blocker, and the LO phase noise mixed onto the blocker. The blocker amplitude affects the last two of these. For our modeling examples, it is most convenient to relate the noise figure degradation to the small-signal gain, which is compressed by the blocker. This relationship is best represented using zero, first and second order terms:

$$\text{Noise Factor} = n_0 - n_1(SSG) + n_2(SSG)^2 \qquad (7)$$

The blocker causes the small signal gain to be reduced while the system noise increases, causing a composite degradation in the noise figure of the system. This is taken into account when the Equations (6) and (7) are fit to actual measurements.

Finally, after the blocker is compressed, the LO noise is reciprocally mixed onto it (see Appendix B), and the noise at the blocker-desired frequency offset falls into the desired band. This noise is then combined with the other contributors.

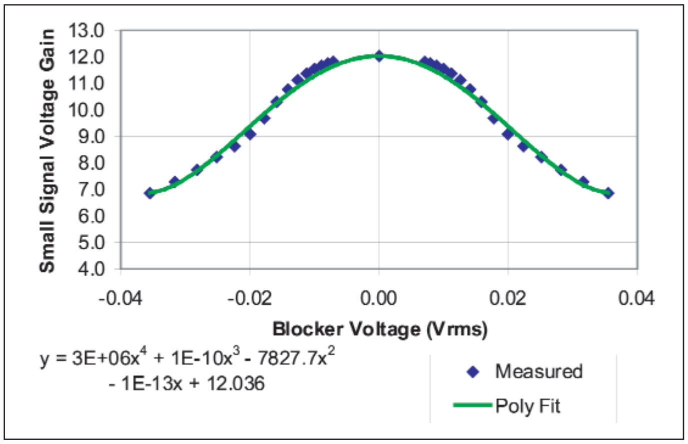
Measurements

Figure 5 shows the test setup. Small-signal gain and NF are both measured on the HP8970B noise figure meter. The blocker generator is filtered so as not to emit noise in the desired band, and the LO generator is filtered so as not to emit noise that would mix with the blocker. Also, the IF output is filtered so the blocker does not hit the NF meter with excessive power. The combiner has isolators on either side of it. On the noise source side, the isolator protects the noise source from the blocker. On the blocker side, the isolator insures that the port sees 50 ohms both inside and outside the passband of the blocker filter.

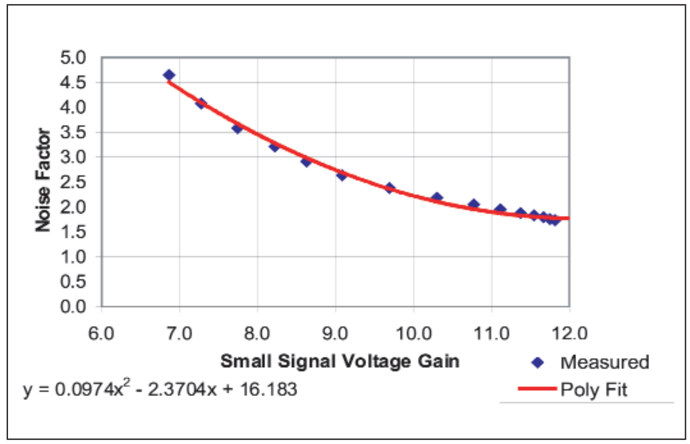
The gain and NF with no blocker is measured, then a blocker is applied and the measurement is repeated for different blocker amplitudes. The measurements are stopped once the blocker reaches the highest value expected in the GSM system, or once the noise figure degrades by more than about 6 to 8 dB. Note that the frequencies of the measured noise band and of the blocker need to be chosen to avoid mixer spurs. If the blocker is located on or near a low-order mixer spur, then the NF meter will register an incorrect measurement.

Generating a model from the data

The gain data is converted to voltage gain and the noise figure data to noise factor, since the polynomial equations need to act upon linear quantities rather than dB. A spreadsheet is used to plot the data and produce best-fit curves. Figure 6 shows the curve for the small-signal gain versus blocker voltage for the Conexant RF212 LNA/mixer in its GSM900 path. The left-hand side of the curve is artificially constructed by folding the data over, resulting in a fit that uses all even coefficients



▲ **Figure 6. Small-signal gain versus blocker voltage.**



▲ **Figure 7. Noise factor versus small signal gain.**

to fit with Equation (6).

In the equation for the best-fit curve, only the even terms are significant because the curve was forced to be even. The orders of the significant terms are zero, second and fourth. The zero-order term is forced to be the same as the no-blocker gain. The equation for this curve defines the blocker dependence of the "small signal gain" box in Figure 4.

Figure 7 shows a plot of the noise factor versus small signal gain for the Conexant RF212 LNA/Mixer. The equation has terms of order zero, first and second. This equation defines (indirectly) the blocker dependence of the "noise figure degradation" box in Figure 4.

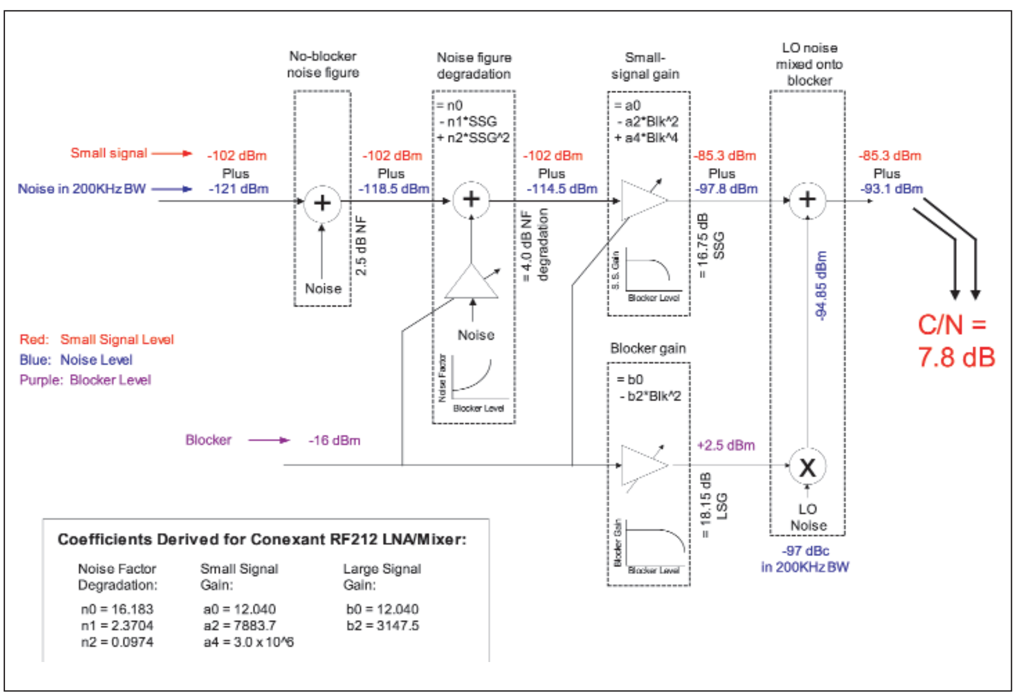
Making use of the model

Once these polynomial equations are generated, the noise contributions are summed up to determine the carrier/noise ratio (C/N) at the LNA/mixer output. An example case is shown in Figure 8, where a desired signal of –102 dBm and a blocker of –16 dBm occur on the LNA/mixer input.

To find the composite C/N, the basic quantities of gain, noise figure and P_{1dB} are required, as are the coefficients of the small-signal gain polynomial (a_0, a_2, a_4), and the noise factor polynomial (n_0, n_1, n_2). The small signal gain and the noise figure can be found based on the level of the blocker, and the reciprocally mixed LO phase noise can be added at the end.

The large signal compression must be applied to the blocker itself, since in our model this compression occurs before the LO noise reciprocal mixing is applied. A polynomial with higher orders can be used for the LSG. It is useful to apply third-order distortion only for the LSG as long as the large signal does not go far beyond the P_{1dB} point of the system. Therefore, the LSG coefficients can be derived either in the same manner as the SSG coefficients, or they can be obtained from the measured P_{1dB} and the relationship in Equation (4) where k_5=0. Appendix C details the P_{1dB} derivation.

For our example, the –16 dBm blocker at the input of the Conexant RF212 LNA/Mixer causes the noise figure to degrade from 2.5 dB — the typical value with no blockers present — to 6.5 dB. The small signal gain drops from 21.6 dB to 16.75 dB. Then,



▲ **Figure 8. Example of model's prediction of C/N.**

the cumulative effects of the compressed SSG and NF result in a 12.5 dB C/N before the LO noise is added and reciprocally mixed onto the blocker which has been amplified by the LSG. The sum total C/N is 7.8 dB.

What makes this approach powerful is that once the model is derived, in can be plugged into the chain calculations for a complete receiver. Then, various what-if analyses can be done with accurate results for any blocker level hitting the LNA/mixer. For example, the effect of different front-end SAW filters with different levels of stopband attenuation for out-of-band blockers can be checked, and an accurate trade-off can be made between the SAW's passband insertion loss and its stopband selectivity.

Furthermore, the approach can be used to accurately estimate the sensitivity level for a receiver in the presence of a blocker. In the typical GSM receiver, the sensitivity level is where the C/N drops to about 6.0 dB. For our example with a –16 dBm blocker, the sensitivity level estimate is –103.8 dBm.

The calculation can also show which contributors are the most important. In this example, even with the seemingly high noise figure of 6.5 dB, the LO phase noise floor of –150 dBc/Hz is still the most significant contributor to the system noise, due to the blocker's presence. If the LO phase noise improves by 1 dB, then the sensitivity improves by 0.7 dB.

Conclusion

This article discussed steps in generating a polynomial model of blocker effects on small-signal gain and noise figure for LNA/mixer devices used in wireless receivers. Based on empirical measurements of gain and noise figure of the device, the polynomials relating gain compression and noise figure degradation to blocker level at the input of device were generated. The polynomial coefficients were then applied to the calculation of carrier-to-noise ratio in the presence of a blocker. The outlined approach makes it possible for a variety of what-if analyses to provide accurate results in predicting blocker-degraded C/N. ■

References

1. W. E. Sabin and E.O. Schoenike, *Single Sideband Systems and Circuits*, Second edition, New York: McGraw Hill, 1995.

2. Behzad Razavi, *RF Microelectronics*, New York: Prentice Hall, 1998.

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Appendix A

Nonlinear analysis for large and small signals

Using a power series expansion, the output of a gain stage can be related to its input as

$$V_0(t) = k_1 v_i(t) + k_2 v_i^2(t) + k_3 v_i^3(t) + k_4 v_i^4(t) + k_5 v_i^5(t) + \ldots \qquad \text{(a1)}$$

Note that if the gain were perfectly linear, then k_1 would be the only nonzero coefficient and the gain would be identical to k_1.

For a two-tone input, $V_i(t)$ is:

$$V_i(t) = A\cos(\omega_1 t) + B\cos(\omega_2 t) \qquad \text{(a2)}$$

Inserting Equation (a2) into Equation (a1) and using the well-known trigonometric equalities, the expression for $V_0(t)$ can be written as:

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$$V_0(t) = \frac{1}{2}k_2 A^2 + \frac{1}{2}k_2 B^2 + \left\{k_1 A + \frac{3}{4}k_3 A^3 + \frac{3}{2}k_3 AB^2 + \frac{5}{8}k_5 A^5 + \frac{15}{4}k_5 A^3B^2 + \frac{15}{8}k_5 AB^4\right\}\cos(\omega_1 t) +$$
$$\left\{k_1 B + \frac{3}{4}k_3 B^3 + \frac{3}{2}k_3 A^2B + \frac{5}{8}k_5 B^5 + \frac{15}{4}k_5 A^2B^3 + \frac{15}{8}k_5 A^4B\right\}\cos(\omega_2 t) +$$
$$\left\{\frac{1}{2}k_2 A^2 + \frac{1}{2}k_4 A^4 + \frac{3}{2}k_4 A^2B^2\right\}\cos(2\omega_1 t) + \left\{\frac{1}{2}k_2 B^2 + \frac{1}{2}k_4 B^4 + \frac{3}{2}k_4 A^2B^2\right\}\cos(2\omega_2 t) +$$
$$\left\{\mathrm{k}_2 AB + \frac{3}{2}k_4 A^3B + \frac{3}{2}k_4 AB^3\right\}\cos((\omega_1 + \omega_2)t) + \left\{k_2 AB + \frac{3}{2}k_4 A^3B + \frac{3}{2}k_4 AB^3\right\}\cos((\omega_1 - \omega_2)t) + \quad \text{(a3)}$$
$$\left\{\frac{1}{4}k_3 A^3 + \frac{5}{16}k_5 A^5 + \frac{5}{4}k_5 A^3B^2\right\}\cos(3\omega_1 t) + \left\{\frac{1}{4}k_3 B^3 + \frac{5}{16}k_5 B^5 + \frac{5}{4}k_5 A^2B^3\right\}\cos(3\omega_2 t) +$$
$$\left\{\frac{3}{4}k_3 A^2B + \frac{5}{4}k_5 A^4B + \frac{15}{8}k_5 A^2B^3\right\}\cos((2\omega_1 \pm \omega_2)t) +$$
$$\left\{\frac{3}{4}k_3 AB^2 + \frac{5}{4}k_5 AB^4 + \frac{15}{8}k_5 A^3B^2\right\}\cos((\omega_1 \pm 2\omega_2)t) +$$
$$\frac{1}{2}k_4 A^3B\cos((3\omega_1 \pm \omega_2)t) + \frac{1}{2}k_4 AB^3\cos((\omega_1 \pm 3\omega_2)t) + \frac{3}{4}k_4 A^2B^2\cos((2\omega_1 + 2\omega_2)t) +$$
$$\frac{1}{8}k_4 A^4\cos(4\omega_1 t) + \frac{1}{8}k_4 B^4\cos(4\omega_2 t) \ldots\ldots\ldots\ldots\ldots$$

The two terms that are of interest are the first-order gain terms. At frequency ω_1, the output voltage is

$$V_0(\omega_1) = k_1 A + \frac{3}{4}k_3 A^3 + \frac{3}{2}k_3 AB^2 + \frac{5}{8}k_5 A^5 + \frac{15}{4}k_5 A^3B^2 + \frac{15}{8}k_5 AB^4 \quad \text{(a4)}$$

Similarly, at $\omega_{2,}$ the output voltage is

$$V_0(\omega_2) = k_1 B + \frac{3}{4}k_3 B^3 + \frac{3}{2}k_3 A^2B + \frac{5}{8}k_5 B^5 + \frac{15}{4}\mathrm{k}_5 \mathrm{A}^2\mathrm{B}^3 + \frac{15}{8}k_5 A^4B \quad \text{(a5)}$$

Dividing both sides of Equations (a4) and (a5) by their respective inputs yields gain at the corresponding frequencies.

$$G(\omega_1) = k_1 + \frac{3}{4}k_3 A^2 + \frac{3}{2}k_3 B^2 + \frac{5}{8}k_5 A^4 + \frac{15}{4}k_5 A^2B^2 + \frac{15}{8}k_5 B^4 \quad \text{(a6)}$$

$$G(\omega_2) = k_1 + \frac{3}{4}k_3 B^2 + \frac{3}{2}k_3 A^2 + \frac{5}{8}k_5 B^4 + \frac{15}{4}k_5 A^2B^2 + \frac{15}{8}k_5 A^4 \quad \text{(a7)}$$

Let us assume A represents the large signal blocker and B the small desired signal. This means A>>B; therefore, we can approximate the above gain terms by letting B go to zero:

$$G(\omega_1) = k_1 + \frac{3}{4}k_3 A^2 + \frac{3}{2}k_3 B^2 + \frac{5}{8}k_5 A^4 + \frac{15}{4}k_5 A^2B^2 + \frac{15}{8}k_5 B^4 \quad \text{(a8)}$$

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$$G(\omega_1) = k_1 + \frac{3}{4}k_3 A^2 + \frac{5}{8}k_5 A^4 \tag{a9}$$

Similarly, we can write:

$$G(\omega_2) = k_1 + \frac{3}{4}k_3 B^2 + \frac{3}{2}k_3 A^2 + \frac{5}{8}k_5 B^4 + \frac{15}{4}k_5 A^2 B^2 + \frac{15}{8}k_5 A^4 \tag{a10}$$

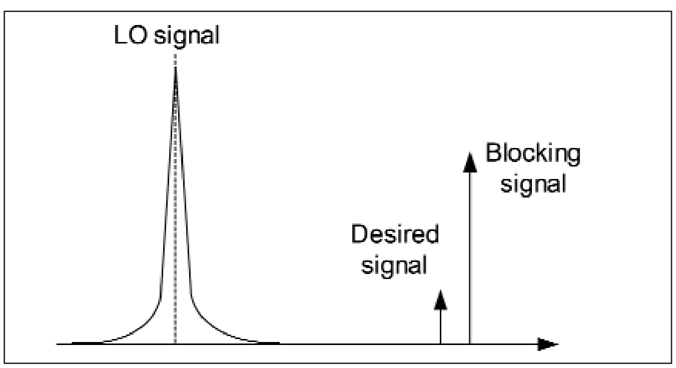
$$G(\omega_2) = k_1 + \frac{3}{2}k_3 A^2 + \frac{15}{8}k_5 A^4 \tag{a11}$$

Equations (a9) and (a11) relate the large signal gain and small signal gain respectively of an amplifier. The interesting point is that both gains depend on the large signal amplitude and, further that under large signal interference, the small signal gain suffers faster. This is apparent by comparing the coefficients of A^2 and A^4 in the above equations.

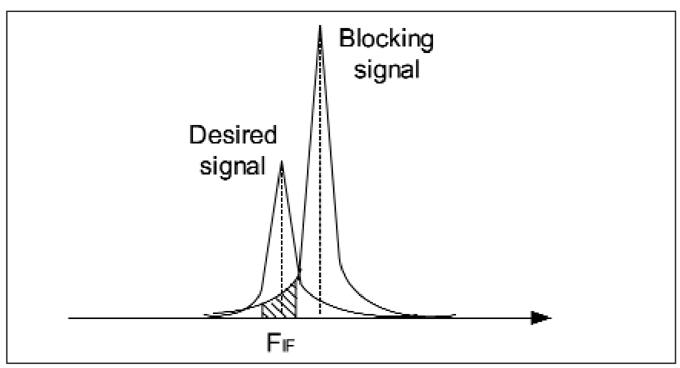
Appendix B

Reciprocal mixing

In a receiver, a small desired signal may be accompanied by a large blocking signal at some frequency offset. The local oscillator that is used to mix the desired channel to IF exhibits finite phase noise at this frequency offset. When the two signals are mixed by the LO, each one has the LO's phase noise spectrum modulated onto it. This process is referred to as "reciprocal mixing." Therefore, the downconverted band consists of two overlapping spectra, with the wanted signal suffering from significant noise overlap due to the tail of the blocking signal's spectrum.



▲ **Figure B(1). RX input signals and LO with phase noise spectrum.**



▲ **Figure B(2). Signals downconverted to IF, with overlap of reciprocally mixed noise.**

Appendix C

P_{1dB} point from third-order coefficient

To obtain the value of single-tone P_{1dB} gain compression induced by only third-order non-linearity, equation (a3) of Appendix A is used, setting higher order terms as well as signal "B" to zero.

$$V_0(\omega_1) = k_1 A + \frac{3}{4} k_3 A^3 + \frac{3}{2} k_3 AB^2 + \frac{5}{8} k_5 A^5 + \frac{15}{4} k_5 A^3 B^2 + \frac{15}{8} k_5 AB^4 \quad \text{(c1)}$$

Setting the higher order terms equal to zero yields

$$V_0 = \left\{ k_1 A + \frac{3}{4} k_3 A^3 + \frac{3}{2} k_3 AB^2 \right\} \quad \text{(c2)}$$

Setting B = 0 and dividing both sides by A yields

$$V_0 = \left\{ k_1 A + \frac{3}{4} k_3 A^3 \right\} \quad \text{(c3)}$$

$$G = \frac{V_0}{\mathrm{A}} = k_1 + \frac{3}{4} k_3 A^2 \quad \text{(c4)}$$

Equation (c4) is the gain with third-order nonlinearity. At P_{1dB}, the overall gain is reduced by 1 dB from the linear gain; that is, the voltage gain becomes $k_1 \times (10^{-1/20})$. To find the amplitude at P_{1dB}, we solve:

$$k_1 + \frac{3}{4} k_3 A^2 = 10^{-\frac{1}{20}} k_1 \quad \Rightarrow \quad A^2 = \frac{4}{3} \left\{ 10^{-\frac{1}{20}} - 1 \right\} \frac{k_1}{k_3} \quad \text{(c5)}$$

Note that "A" represents the amplitude at which P_{1dB} occurs.
For easier manipulation, we set

$$\alpha = \frac{4}{3} \left\{ 10^{-\frac{1}{20}} - 1 \right\} \quad \text{(c6)}$$

Then,

$$A = \sqrt{\alpha \frac{k_1}{k_3}} \quad \text{(c7)}$$

Equation (c7) represents the amplitude at which gain is compressed by 1 dB.

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To find 1 dB compression point in terms of power, we begin with Equation (c3).

$$V_0 = \left\{ k_1 A + \frac{3}{4} k_3 A^3 \right\}$$

Next, we raise both sides to power 2.

$$V_0^2 = \left\{ k_1 A + \frac{3}{4} k_3 A^3 \right\}^2 \quad \text{(c8)}$$

After some routine arithmetic and replacing A with its equivalent α, we obtain

$$V_0^2 = \left\{ \alpha + \frac{3}{2}\alpha^2 + \frac{9}{16}\alpha^3 \right\} \frac{k_1^3}{k_3} \quad \text{(c9)}$$

Now, α can be numerically evaluated as

$$\alpha = \frac{4}{3}\left\{ 10^{-\frac{1}{20}} - 1 \right\} = -0.145$$

And V_0^2 becomes

$$V_0^2 = -0.11518 \frac{k_1^3}{k_3} \quad \text{(c10)}$$

Now V_0 is a peak voltage (we will call it V_p). Converting to dBm yields

$$P_{dBm} = 10\,\text{Log}\left\{ \frac{V_p^2}{2R} 1000 \right\} = 10\,\text{Log}\left\{ V_p^2 \frac{50}{R} 10 \right\} \quad \text{(c11)}$$

where R is the system source resistance.

Inserting (c10) into (c11) gives P_{1dB} in terms of the amplifier's coefficients.

$$P_{1dB} = 10\,\text{Log}\left\{ -0.11518 \frac{k_1^3}{k_3} \frac{50}{R} 10 \right\} \quad \text{(c12)}$$

Since an amplifier's k_3 is a negative term, its sign can be absorbed, and the term can be presented in the absolute form, which then results in

$$P_{1dB} = 10\,\text{Log}\left\{ \frac{k_1^3}{|k_3|} \frac{50}{R} \right\} + 0.614 \quad \text{(c13)}$$

In a 50-ohm system, the R becomes obsolete and P_{1dB} is reduced to

$$P_{1dB} = 10\,\text{Log}\left\{ \frac{k_1^3}{|k_3|} \right\} + 0.614 \quad \text{(c14)}$$