Packing More Antenna into Available Space

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Wireless applications, particularly with multiple resonances, put new demands on antennas pertaining to size, gain, efficiency, bandwidth, and more. One promising approach in this regard is to use fractal geometries to find the best distribution of currents within a volume to meet a particular design goal. Within this world of complex geometries, engineers need the most efficient method using the most effective tool.

CST MICROWAVE STUDIO® (CST MWS) from Computer Simulation Technology is a time-domain tool capable of analyzing broad-band structures with multiple resonances. It was used to study these fractal geometries using the example of a Sierpinski Triangle Antenna. The simulation demonstrated the ability of CST MWS to match the measured response of this broadband device in a single simulation, while also providing control over complex geometry construction with the built-in VBA macro editor.
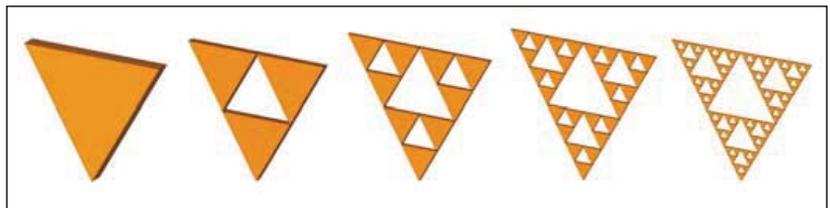
Sierpinski Triangle antenna

A fifth iteration Sierpinski Gasket (Triangle) Monopole was simulated using the finite integration technique (FIT) [1]. Five resonances are clearly shown, illustrating that the number of resonances for this antenna increases with the band number of this fractal shape [2]. Surface currents demonstrated the published patterns mimicking frequency dependent arrays of bow-tie elements. Simulated results for S_{11} magnitude matched measured results within anticipated normal variations across the band.

The first reference to having used a Sierpinski Triangle (or gasket) as an excited antenna was made in 1995 [3] by Cohen. A true Sierpinski Triangle is not a surface, but starts with an initiator triangle and a generator triangle. The generator is applied, in its various forms, to the initiator to infinity. The equations governing the mapping of the three vertices from one two-dimensional space to another two-dimensional space are of the form [4]:

$$s_1(x,y) = \left(\frac{x}{2}, \frac{y}{2}\right)$$
$$s_2(x,y) = \left(\frac{x+1}{2}, \frac{y}{2}\right) \qquad (1)$$
$$s_3(x,y) = \left(\frac{2x+1}{4}, \frac{2y+\sqrt{3}}{4}\right)$$

In reality, such equations would lead to 19,683 points, with some redundancy, by the ninth iteration! A much simpler way to do it would be to use progressively larger groupings. In this way, the complexity of building such a shape could be greatly simplified (see Figure 1).

Structures up to ninth iteration have been built, but not analyzed, using 6,561 elements.

▲ **Figure 1: Five stages of making a fifth iteration Sierpinski Triangle. The first iteration (band number equals one) is formed by the solid bow-tie triangle. Subsequent images are scaled by 1/2.**
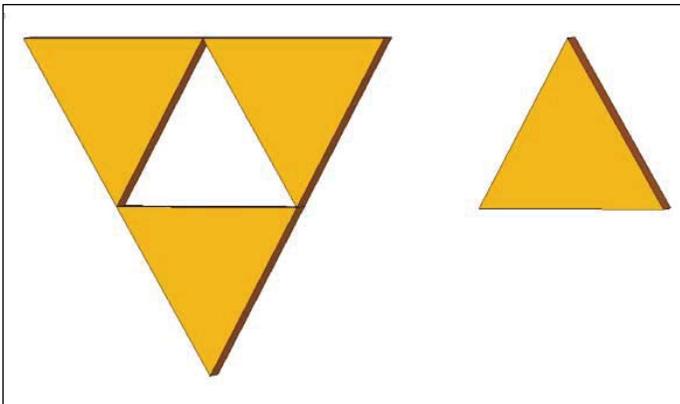
▲ **Figure 2. The second iteration result (left) of subtracting the generator triangle (right) from the initiator triangle.**

Fractal dimension has been described as [4]:

$$D_s = \frac{\ln(N)}{-\ln(\gamma)}$$

where N = non overlapping copies of the whole scaled by γ. In this case, going back to the original definition of the structure, $N = 3$ with $\gamma = \frac{1}{2}$ for a dimension of 1.585.

That means, in the pure case with the generator triangle being applied to the initiator triangle to infinity (see Equation (1) and Figure 2) the self-symmetry is three-fold and the scaling is $\frac{1}{2}$. It has been suggested that since

$$\frac{\partial \vec{J}}{\partial t} \rightarrow radiation$$
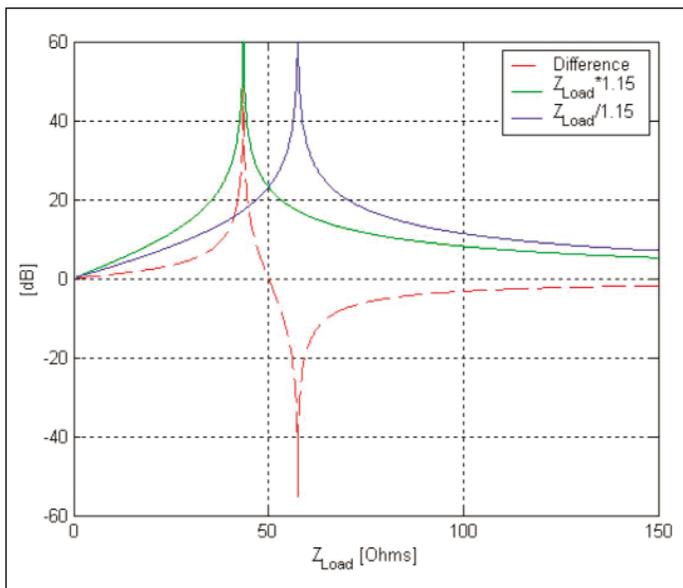
▲ **Figure 3. Measurement sensitivity showing S_{11} variation due to ±15 percent uncertainty in the load resistance.**

there may be a connection between the dimension of a fractal, and its efficiency as a radiator, but this has never been quantified. It has been shown, however, qualitatively valid. Also, the Euclidian formulation of the minimum radiation Q may not apply to fractals [4]. The Euclidian formulation for this goal could be taken as that expressed by the following equations. The minimum radiation Q for circularly polarized waves [5]:

$$Q = \frac{1}{2}\left(\frac{1}{k^3 a^3} + \frac{2}{ka}\right)$$

and the minimum radiation Q for linear waves:

$$Q = \frac{1}{k^3 a^3} + \frac{1}{ka}$$

where k is the wavenumber

$$\left(\frac{2\pi}{\lambda}\right)$$

and a is the radius of a sphere superscribing the antenna. In general, "the relationship between radiation Q and maximum achievable bandwidth is not straightforward" [5]. However, if $Q >> 1$, then BW = f / Q is a good approximation. If one is so successful that $Q >/> 1$, or even $Q < 1$, then this simple relation does not hold.

Once it has been constructed on its substrate, and a ground-plane added beneath, the resulting antenna is fed between the lower vertex and ground, with reflection coefficient results similar to a bow-tie monopole of the same overall dimensions, or individual subgasket dimensions [2]. By the fifth iteration there are 81 triangles, 123 vertices and 243 edges to solve, with more than four wavelengths in the longest dimension.

Modeling the antenna

Figure 3 illustrates measurement sensitivity. A ±0.15 variation in input resistance yields an infinite difference in S_{11} values, limited in the plot to 60 dB maximum by the resolution of the calculation.

At the higher frequencies such as 16 GHz on an antenna with a fundamental in the MHz range, second-order effects become significant. There are many unknowns at the feedpoint, such as edge-effects, parasitics, feed-geometry, and so forth, and the fact that these are not quantified does not mean they are not influencing the result.

Comparing measured results

Getting to the measured data, a fifth iteration fractal antenna was published in the *IEEE Transactions on*

Antennas and Propagation [2]. See Figure 4 for these results as compared to the results from CST MWS.

It seems significant that the number of resonances increases with the number of iterations. The definition for first iteration here is the same used by Mittra, Puenta, et al. In some ways, it would make more sense to define the zeroth iteration as the one prior to the application of any generator to the initiator, eschewed here for consistency. The scale between resonances is approximately 2, except between the lowest two frequencies. It is thought that this difference is due to self-symmetry truncation at the low end [2]. This means that the structure is symmetric with respect to itself, or looks the same at any and all scales that fill the field of view as the order increases. But, as the order decreases, the fractal looks less and less like itself, until the second and first iterations are compared. Self-symmetry about the feedpoint along with origin symmetry — consider the dipole form — have been shown to be requirements for frequency independence [6]. Broadbandedness is another issue, defined by self-complementarity [1]. To go into this subject any further would be beyond the scope of this short article.

The measured antenna was made of copper-clad substrate with a relative permittivity of 2.5. The CST MWS simulation was made using solid perfect electric conductor (PEC) on a substrate with the same permittivity. Also, the measured antenna was on an 800×800 mm flat metal plate [2], while the CST MWS simulation was run with an infinite ground plane. These differences created a faster simulation. Using copper for the metal in both the antenna and the ground plane did not appreciably change the result. There did not seem to be a need for increasing the run-time in the absence of values for specific structural changes to the feed geometry. It was noticed during simulation that the antenna driving point impedance could be "tuned" by altering the gap between the feed point at the lower vertex, and ground. A final value of 1 mm was used with good results.

An important point may be observed within the example of Figure 5. Although in the original article [2] the antenna was not fed at its driving point impedance (ca.150 [Ohms]), nevertheless above a certain lower-limit frequency, the impedance remains within the confines of a circle indicating that the antenna is becoming very broad-band. By adjusting the driving point impedance, it is possible to move this circle into concen-tricity with the origin. The maximum attendant VSWR would be around 3, in this case.

The current distribution at 3.51 GHz has been plotted (see Figure 6). In this case the current maxima appear as would be expected for the third harmonic
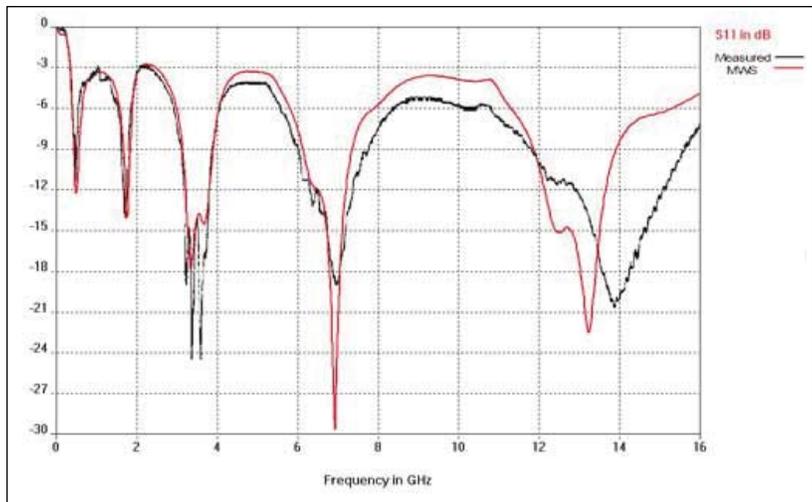

▲ **Figure 4. S_{11} simulated versus measured data.**

(with the first harmonic defined as equal to the fundamental). In the corresponding radiation pattern, the characteristic three-lobed farfield pattern of an impedance-matched monopole is displayed, as expected [2] (see Figure 7).

Conclusion

A fifth iteration Sierpinski Gasket (Triangle) Monopole was simulated with an FIT-based code (Finite Integration Technique) using CST MICROWAVE STUDIO, which is capable of analyzing broad-band structures with multiple resonances. The simulation demonstrated the ability of CST MWS to match the measured response of this broadband device in a single simulation, while also giving the engineer control over complex
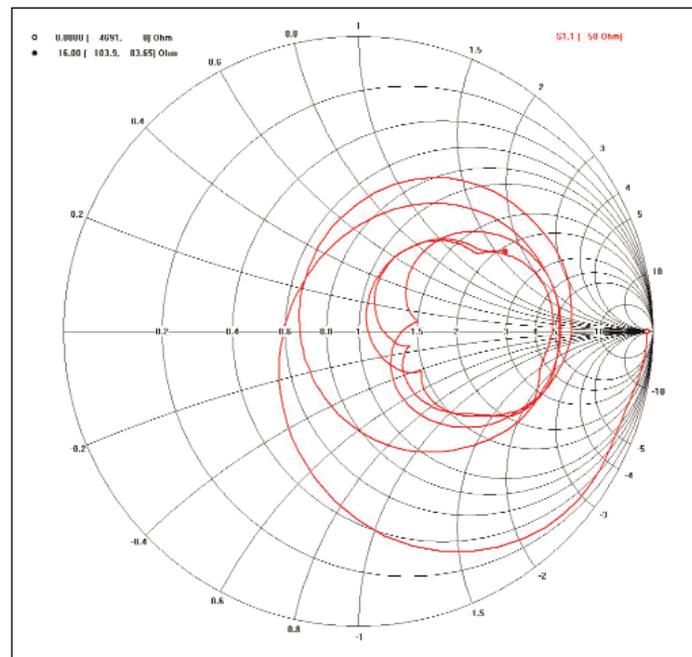
▲ **Figure 5. The simulated Smith Chart impedance.**
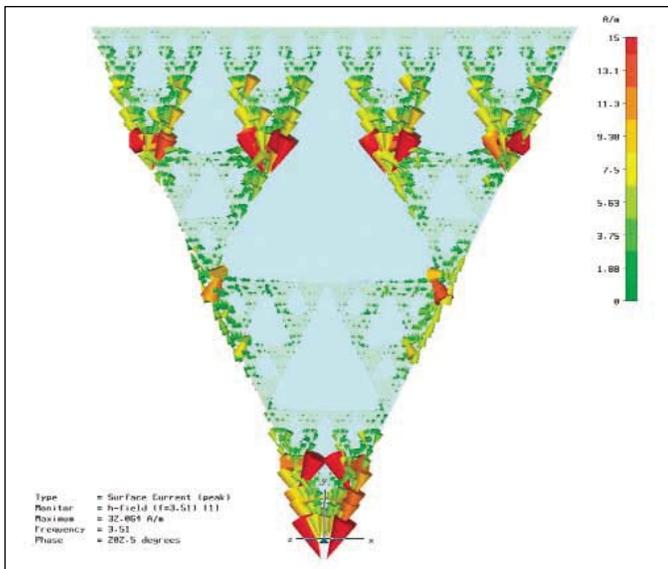
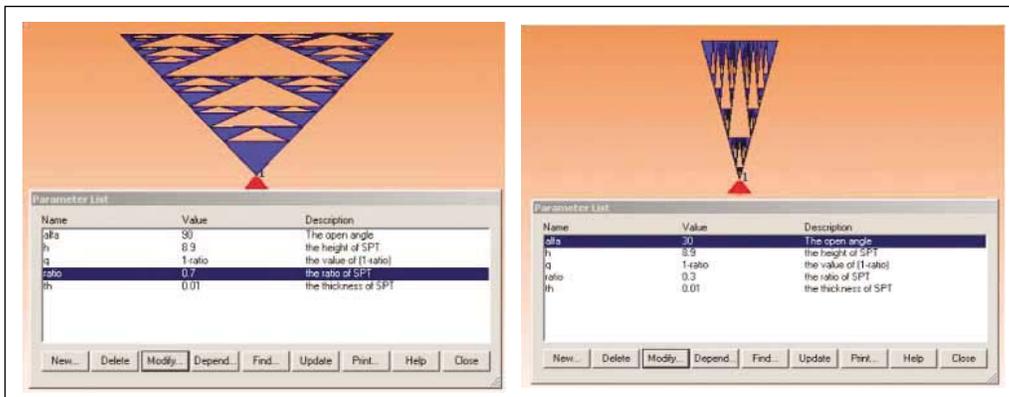▲ **Figure 6. Current distribution.**
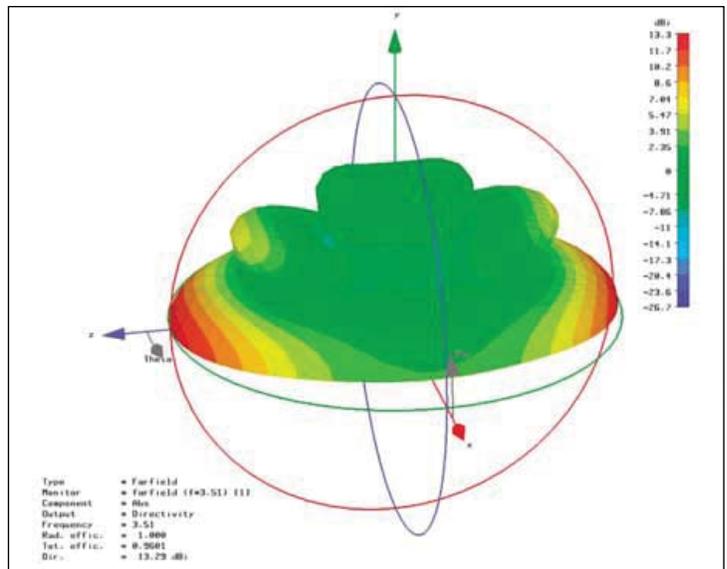
▲ **Figure 7. Farfield pattern.**

geometry construction with the use of the built-in VBA macro editor. The use of a structure made up entirely of triangles proves that the Perfect Boundary Approximation® (PBA) technique (see below) works very well in this case. The far-field pattern shows the expected three-lobed shape.

About CST MICROWAVE STUDIO

CST MICROWAVE STUDIO uses FIT, a one-to-one translation of Maxwell's equations into a discrete space formulation without simplification or specialization. This theoretical foundation has been developed dynami-

cally over more than 25 years. The explicit time domain approach is particularly well suited to this type of antenna. The mesh cell pattern is strictly rectangular 3-D, non-uniform and orthogonal. Typical errors introduced by staircase meshing are avoided through the system's Perfect Boundary Approximation® (PBA), which in CST MWS's Version 4 allows for "split cells" in which the fields on either side of a good conductor are isolated by the metal layer. The boundary between two dielectrics, or between a dielectric and a conductor, can be found by this method without resorting to other high-overhead meshing methods.

The user interface is based on the latest ACIS-kernel, making it as easy to enter structures as with a CAD program. CST MWS employs the VBA language (Sax Basic) comprising the standard language elements along with a couple of CST-specific language extensions. These macros are very powerful aides to the simulation engine, basically in two ways. Control tasks, used for simulation controls, are not stored

▲ **Figure 8. Illustrations of vertical scaling and variations in the feed angle showing a user-friendly parameterization of the model.**

in the history list. The history list is a complete description of the structure, boundary conditions, symmetry planes, frequency range of interest, meshing parameters, in short, simulation inputs. Structure modeling tasks, on the other hand, are stored within the history list. Such a macro is illustrated below, and can be used to generate useful variants.

Various schemes have been tried for skewing the resonant frequency, giving the designer more control over the result [4]. A straightforward macro can be written to reproduce these designs, which vary either in vertical scaling or included angle at the feed point. By combining these two variables into one macro, both can be varied at will reproducing any combination within reasonable bounds (see Figure 8). This macro is available to customers and testers upon request. ■
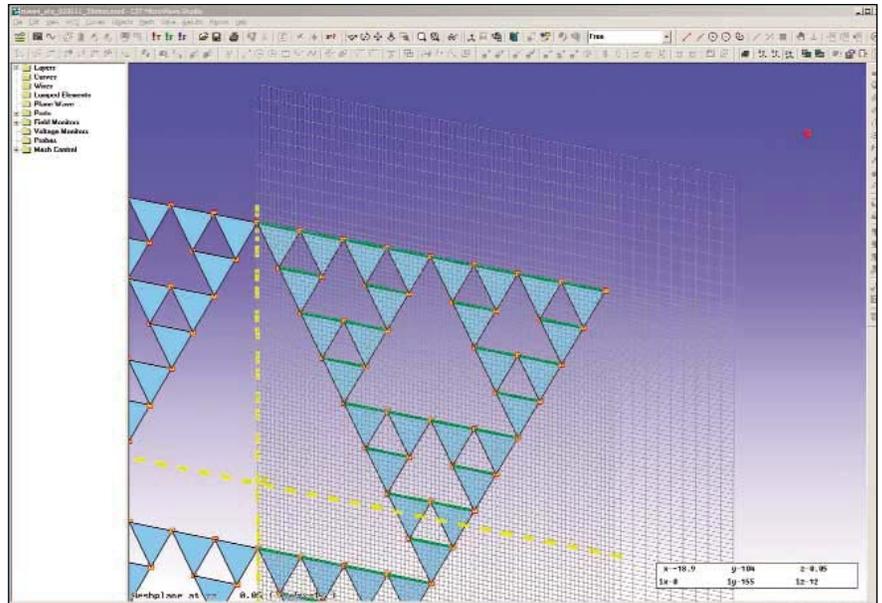
▲ **Figure 9. The mesh and user interface of CST MICROWAVE STUDIO using magnetic symmetry.**

Acknowledgements

References:

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2. C. Puenta, J. Romeu, R. Pous and A. Cardama, "On the Behavior of the Sierpinski Multiband Fractal Antenna," *IEEE Transactions On Antennas and Propagation*, 46 (4), April 1998.

3. Nathan "Chip" Cohen, "Fractal Antennas, Part 1," *Communications Quarterly*, Summer 1995.

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6. Robert G. Hohlfeld and Nathan Cohen, "Self-similarity and the Geometric Requirements for Frequency Independence in Antennae," *Fractals*, 7 (1), March 1999.

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