Using Nyquist Stability Analysis to Verify Stability of Wireless Circuits

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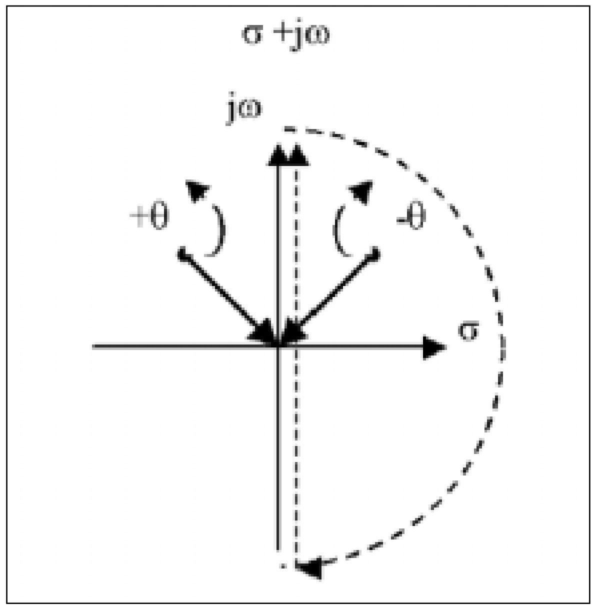
Engineers designing RF circuits for optimum stable performance have come to rely on the standard methods when determining the stability of a two-port network. While these methods are theoretically sound, they apply only to linear two-port networks. As a result, they cannot predict circuit stability in many cases, especially those in broadband wireless applications.

As an alternative, a Nyquist stability analysis is a rigorous method for determining whether a circuit is stable. Although it is a complex approach, Nyquist analysis has been effectively implemented in some electronic design automation (EDA) circuit simulators, which makes it much more attractive for use by designers.

Deriving a method

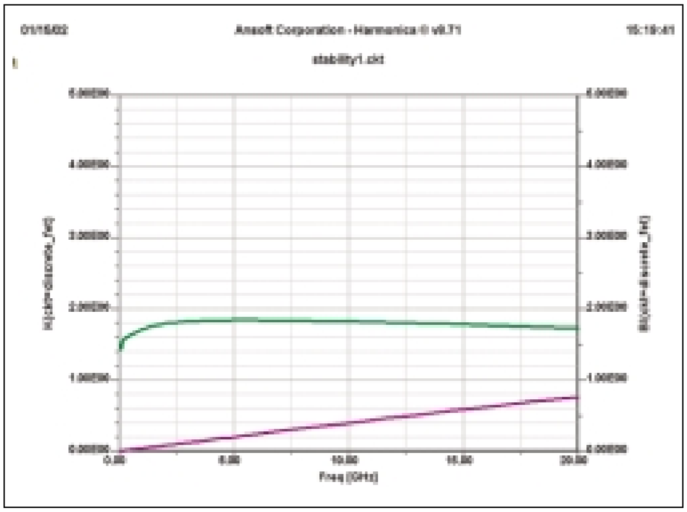
The commonly used stability criteria of $K > 1$ and $|\Delta s| < 1$ (or $B_1 > 0$) are derived from a small-signal, two-port *S*-parameter representation. Because *S*-parameters are defined for steady-state linear networks, nonlinear behavior and non-steady state behavior cannot be detected by this approach.

In contrast, rigorous stability analysis can detect the natural frequencies in the characteristic system equation that occur in the right half of the complex $\sigma + j\omega$ plane (Figure 1). Because natural frequencies give rise to transient time responses ($e^{\sigma t}\sin\omega_0 t$), those in the left half of the plane, $s < 0$, dampen out in time. However, the frequencies in the right half of the plane do not, resulting in an unstable circuit. Formal Nyquist analysis performs an integration of the characteristic system equation in the complex plane. The path of integration begins at zero, proceeds to $0 + j\infty$, follows a semicircle of infinite radius to $0 - j\infty$ and continues along the imaginary access to zero.

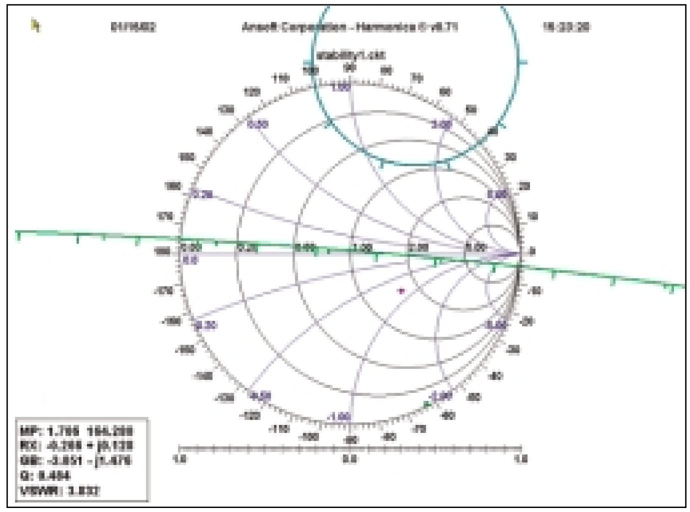


▲ **Figure 1. The Nyquist integration path.**

Any natural frequencies in the right half of the plane will be enclosed by the path of integration; those in the left half will not. In an EDA environment such as Serenade from Ansoft Corporation, the Nyquist analysis is performed by computing the determinant of the closed-loop system from DC to a high frequency and plotting the result on the complex plane. If the path crosses the negative real axis and encircles the origin, then a natural frequency exists in the right-half plane and the system is asynchronously unstable. The term "asynchronous" is used here to indicate that the new frequency the circuit may generate is different from its excita-



▲ **Figure 2. The K1 and B1 of the example transistor.**



▲ **Figure 3. S_{11}, S_{22} and stability circles at 880 MHz.**

tion frequency. In performing the Nyquist analysis, it is especially beneficial to examine the broadband stability of an RF circuit, including the effect of bias networks.

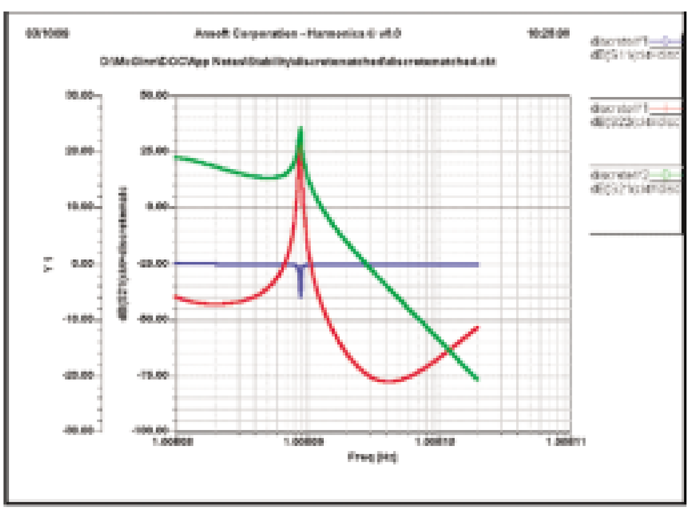
Determining the frequency sweep points from DC to a "high" frequency can be a difficult task if specified by a user. A frequency step too large may miss a sharp resonant feature and a step too small may take excessive computing time. Serenade makes this step simple by dynamically changing the sweep step based on the characteristics of the frequency response so small steps are taken where the response is changing rapidly and large steps are used where the response is changing slowly. The user can specify the maximum step size, if desired.

Circuit analysis

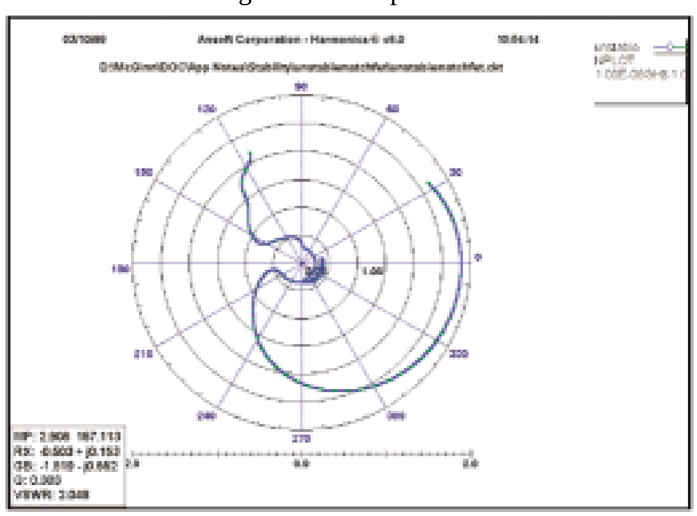
A simple amplifier circuit can be used to illustrate the failure of K1 and B1 to accurately determine stability. In this case, the amplifier is actually stable, while the results of K1 and B1 show otherwise. The circuit's response can be analyzed using small-signal conditions to determine linear stability criteria and design impedance-matching networks. Nyquist analysis can also be used to demonstrate whether or not natural frequencies are present in the right-half plane.

The large-signal parameters for the example are derived from a 400-μm pHEMT and applied to a modified-Materka model that was selected from Serenade's nonlinear active device parts list. The remainder of the circuit consists of a bias network for the gate and drain and RF input and output ports. The software's small-signal AC analysis function was used to obtain the network parameters. In Figure 2, the K-factor is less than 1 below 14 GHz, which results in areas of the "stability" circles intersecting $\rho < 1$ (a positive real impedance).

The source and load stability circles, as well as S_{11} and S_{22} of the device, are shown in Figure 3 for a design frequency of 880 MHz. The "tick marks" on each circle are the "stable" regions and represent a locus of termi-



▲ **Figure 4. Characteristics of the device with input impedance matching.**



▲ **Figure 5. The polar Nyquist plot of the amplifier.**

nations that will result in a positive real circuit impedance at the other port. A negative real impedance would produce a return loss greater than 0 dB, but not necessarily an unstable circuit. In this example, the complex conjugate of the device's input impedance lies inside a region that produces a negative real impedance at the other device port. A simple matching structure derived using Serenade's Smith tool utility allows the circuit to be examined with the new matching circuits. The new response is plotted in Figure 4.

This is obviously an undesirable amplifier re-sponse, and according to the linear stability criteria (K < 1, B1 < 0), it is unstable as well. However, the results of the Nyquist analysis tell a different story. The output of the analysis is the normalized determinant of the harmonic balance system equations and can be plotted on a polar or magnitude-angle graph. Small circuits like this one are best analyzed by viewing the polar output and identifying clockwise encirclements of the origin. Results of the Nyquist analysis (Figure 5) show that, in direct contrast to the linear stability criteria, there is no clockwise encirclement of the origin, and the circuit is in fact stable when terminated in 50 ohms.

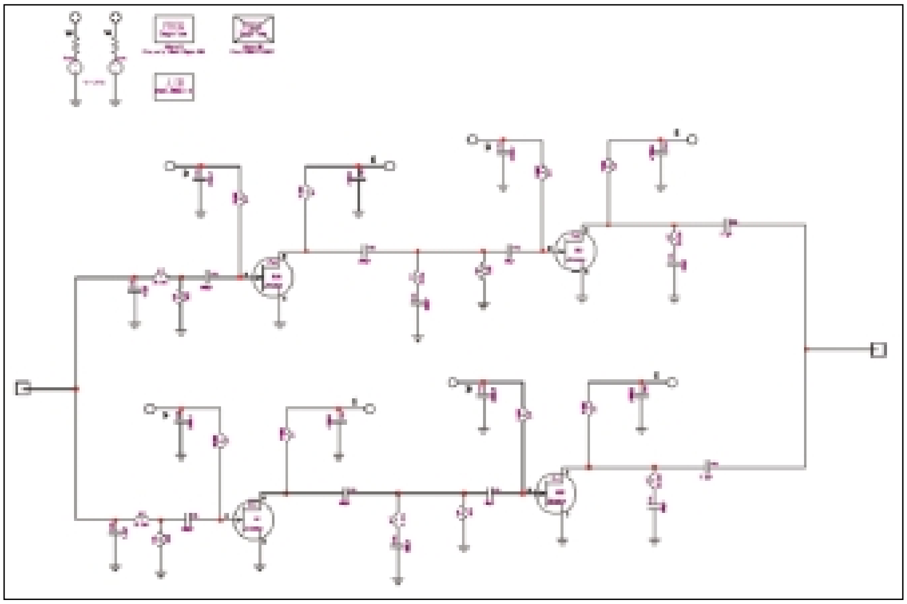
The software's oscillator design aid can be employed to verify these findings. To do this, the frequency control block is modified to allow the simulator to conduct a search for oscillations over a specified frequency range. The results of confirm that conditions required for oscillation to occur are not present in this circuit.

An amplifier example

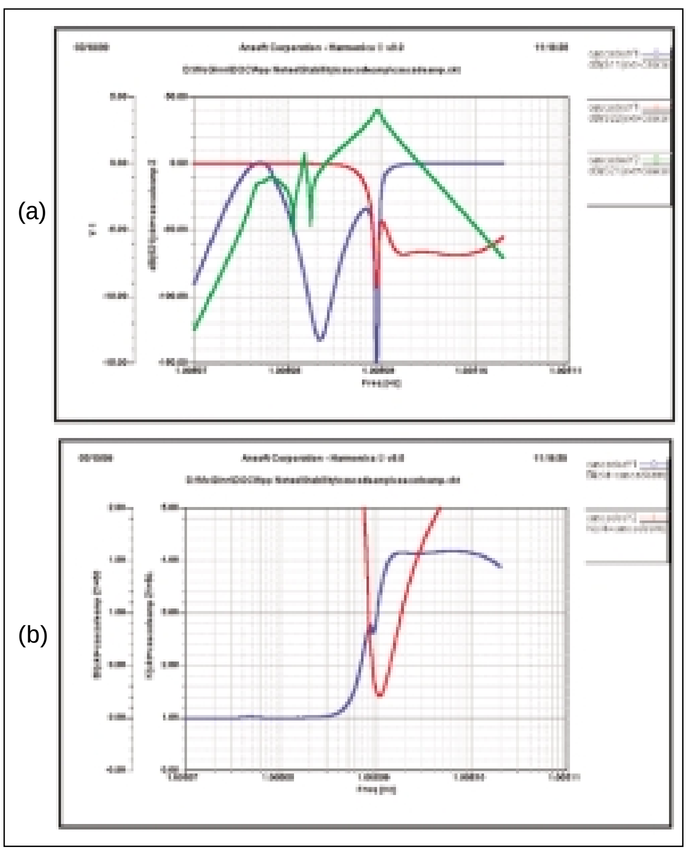
A two-stage amplifier provides another example of the value of the Nyquist analysis. For this discussion, the amplifier has a 120-ohm shunt resistor placed on the gate of the transistor to make it "unconditionally" stable at the design frequency (k > 1, B1 > 0). The sample amplifier topology is composed of dual two-stage cascaded amplifiers in parallel (Figure 6).

The simulation software's Smith tool utility was used to derive the input and output match as well as the interstage matching network. Simulation of the small signal response produced the results shown in Figure 7(a) and 7(b). The results indicate a stable amplifier (k > 1 and B1 > 0) with the gain and return loss specified by the design parameters. If the analysis stopped here, this design would likely be prototyped.

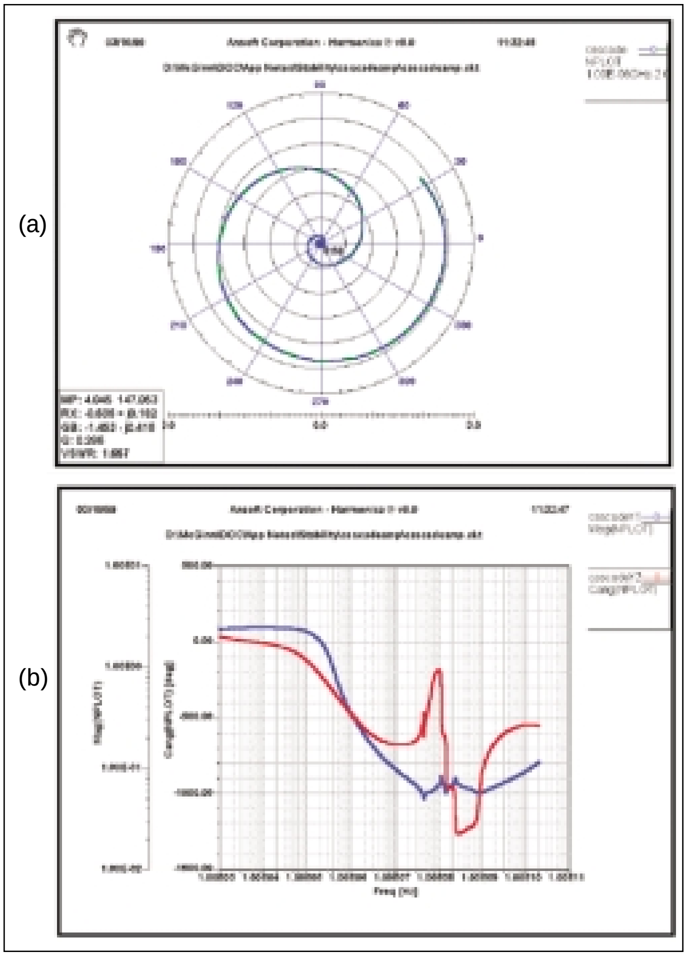
Using the Nyquist analysis, the determinant of the closed-loop system is shown in a polar plot, Figure 8(a), which calls into question the stability conclusion of previous analysis. Yet from this plot, it is not possible to know if the determinant is encircling the origin in a clockwise or counter-clockwise direction.



▲ **Figure 6. Sample amplifier topology composed of dual two-stage cascaded amplifiers in parallel.**

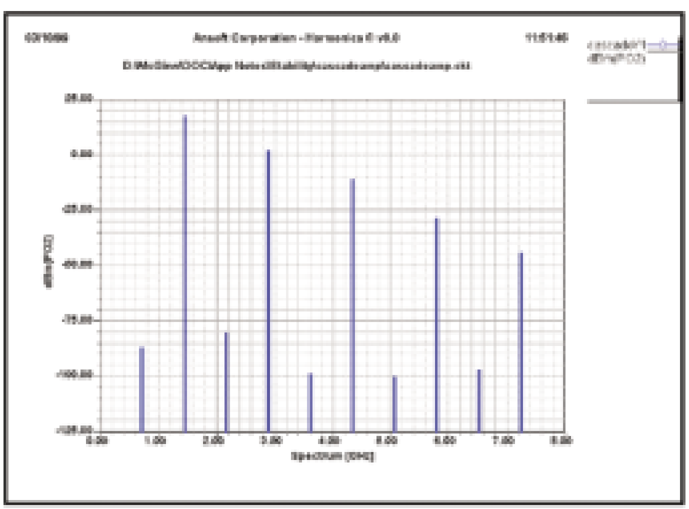


▲ **Figure 7. (a) Small-signal and (b) linear stability criteria of the two-stage amplifier.**



▲ **Figure 8. (a) Polar Nyquist plot of an unstable two-stage amplifier and (b) a magnitude/cumulative angle Nyquist plot of the amplifier.**

A clockwise encirclement of the origin still might not result in instability if the determinant changes direction and "unravels;" a rectangular plot could show if the cumulative angle of the system determinant crosses and remains below –360 degrees, which would confirm that instability exists. Figure 8(b), a magnitude/cumulative angle Nyquist plot, clearly illustrates that the circuit is unstable, despite the predictions of the linear stability criteria.



▲ **Figure 9. Examination of the voltage versus time appearing at the drain of the second-stage transistors revealed two waveforms 180 degrees out of phase.**

For further confirmation, nonlinear oscillator analysis was used to simulate the output spectrum of this amplifier in the absence of RF drive. The oscillation frequency was found to occur in the vicinity of the natural frequency as determined by the frequency where the system equation crosses the negative real axis of the Nyquist polar plot. The output spectrum demonstrates a cancellation of odd harmonics, which is the product of the two output transistors. The examination of the voltage versus time appearing at the drain of the second-stage transistors revealed two waveforms 180 degrees out of phase is shown in Figure 9.

Conclusion

The stability criteria $K > 1$ and $|\Delta s| < 1$ (or $B_1 > 0$) can be inadequate for determining circuit stability, especially in broadband wireless circuits. Although Nyquist stability analysis can be an arduous task, its implementation in EDA tools makes this rigorous method simple to use while providing fast, accurate results. Applying this analysis during the design cycle can improve the chances of a successful prototype, with a commensurate reduction in cost and reduced time to market. ■

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